CONGRUENCES MODULO POWERS OF 2 FOR THE SIGNATURE OF COMPLETE INTERSECTIONS

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In this note we give formulas for the signature of complete intersections modulo certain powers of 2. Our method also yields relations modulo powers of 2 on the signature of ramified covers. We conclude with a formula for the behavior under ramified cyclic covers of Rokhlin's invariant for characteristic surfaces in a 4-manifold.

1. Signatures of intersections

Let $X_{2m}(\underline{d})$ be a nonsingular complete intersection of hypersurfaces of degrees $\underline{d} = d_1, \ldots, d_r$ in CP_{2m+r}. Suppose $d_1, \ldots, d_s$ are even and $d_{s+1}, \ldots, d_r$ are odd integers. Let $d = d_1 \cdots d_r$ denote the total degree. Then the signature of X satisfies the congruence ([**4**] or [**5**])

$$\operatorname{Sign} X_{2m}(\underline{d}) \equiv \begin{cases} d \bmod 8 \text{ if } \binom{m+s}{s} \text{ is odd,} \\ 0 \bmod 8 \text{ if } \binom{m+s}{s} \text{ is even.} \end{cases} \tag{1}$$

This is a simple consequence of the theory of integral quadratic forms. If $\binom{m+s}{s}$ is even, then the intersection form on $H = H_{2m}(X, Z)$ has even type, [**5**, (2.1)]. If $\binom{m+s}{s}$ is odd, the homology class $h \in H$ defined by intersection with a generic CP_{m+r} is characteristic, that is $x \cdot x \equiv x \cdot h \bmod 2$ for any $x \in H$. But also $h \cdot h = d$, so (1) follows from the lemma of van der Blij.

For a complete intersection of complex dimension 2 Rokhlin's formula relating the signature of an oriented 4-manifold and the self-intersection of a characteristic submanifold implies a congruence modulo 16. Namely,

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using the formula for the computation of an Arf invariant in coverings, see § 7, we obtain for complete intersections of odd degree d

$$\operatorname{Sign} X_2(\underline{d}) \equiv d + 8\varepsilon(d) \bmod 16 \tag{2}$$

where

$$\varepsilon(d) = \begin{cases} 0 & \text{if} \quad d \equiv \pm 1 \bmod 8 \\ 1 & \text{if} \quad d \equiv \pm 3 \bmod 8. \end{cases}$$

S. Ochanine [**6**] has generalized Rokhlin's formula to higher dimensions and used mod 16 signatures to compute the Kervaire invariant of certain complete intersections.

These examples suggest the problem: modulo which power of 2 does the signature depend only on the total degree and what are explicit formulas? In this note we give such formulas modulo 64 and higher powers of 2. In particular we show (2) holds for any complete intersection of odd degree d and dimension $2m$ congruent to 2 modulo 4. Our main result is

THEOREM 1. *Let $X_{2m}(\underline{d})$ be a complete intersection as above. Let $D = d_{s+1} \cdots d_r$ be the product of the odd entries in the multidegree, s the number of even entries, and s^* the number of d_i divisible by 4. Then*

$$\operatorname{Sign} X_{2m}(\underline{d}) \equiv d(\alpha_{m,s} - (D^2-1)\beta_{m,s} - 4s^*\gamma_{m,s}) \bmod 2^{\max\{6, 5+s, 4+s+s^*\}} \tag{3}$$

where $\alpha_{m,s}$, $\beta_{m,s}$, and $\gamma_{m,s}$ are constants depending only on m and s, and are given by the generating functions (14) *below.*

The proof will start from the generating function for $\sum \operatorname{Sign} X_{2m}(\underline{d}) z^{2m}$. The result (3) can also be given by a generating function (13); the point is that working modulo an appropriate power of 2 the coefficients are much easier to analyze.

In the following corollaries we give several cases where the congruence has a simpler expression.

COROLLARY 1. *If d is odd,*

$$\operatorname{Sign} X_{2m}(\underline{d}) \equiv d\left(1 + (d^2-1)\left(m - 4\left[\frac{m+1}{2}\right]\right)\right) \bmod 64 \tag{4}$$

where $[r]$ denotes greatest integer $\leq r$.

Note that since, for odd d, 8 divides d^2-1, we need only know $[m+1/2] \bmod 2$; it is 1 for $m = 1, 2 \bmod 4$ and 0 for $m \equiv 3, 4 \bmod 4$.

COROLLARY 2. *Sign $X_{2m}(\underline{d})$ modulo 32 depends only on m, s, and d except when $d \equiv 2 \bmod 4$ when it depends also on D.*

COROLLARY 3. *Modulo* 16 *the signature is determined by* m, s, *and* d *and is given by the generating function*

$$\sum_{m=0}^{\infty} \operatorname{Sign} X_{2m}(\underline{d}) z^{2m} = \frac{d}{(1-z^2)(1+z^2)^s} + \varepsilon(d) \frac{8z^2}{1+z^4} \bmod 16 \tag{5}$$

where $\varepsilon(d) = 0$ *unless* $d \equiv \pm 3 \bmod 8$.

s	m	Sign $X_{2m}(\underline{d})$ mod 16
0	0 mod 2	d
	1 mod 2	$d+8\varepsilon(d)$
1	0 mod 2	d
	1 mod 2	0
2	0 or 5 mod 8	d
	1 or 4 mod 8	$-d$
	2 or 3 mod 8	$2d$
	6 or 7 mod 8	0
3	0 mod 4	d
	1, 2, or 3 mod 4	0
≥ 4	all m	0

2. Real varieties

In [8] Rokhlin showed that the signature of a projective variety coincides modulo 16 with the Euler characteristic of the set of its real points provided they form an M-manifold. Thus Corollary 3 provides simplified formulas for the Euler characteristic mod 16 in the case of complete intersections.

Let $X_n(\underline{d})$ be defined by polynomials with real coefficients and let $A = X \cap RP^{n+r}$ be its set of real points. It is a consequence of Smith theory [8, § 2] that

$$\dim H_*(A; Z/2) \leq \dim H_*(X; Z/2).$$

A is called an M-manifold when equality holds (an M-curve in the classical case $n = 1$ of Hilbert's sixteenth problem).

COROLLARY 4. *Let* $A^{2m}(\underline{d})$ *be an* M-*manifold, then the Euler characteristic* $e(A)$ *is given modulo* 16 *by the table of Corollary* 3.

3. Ramified covers

The method of proof of Theorem 1 also yields information about the signature of ramified covers. Let N be an oriented manifold of dimension divisible by 4 on which the cyclic group $G = Z/d$ acts preserving orientation and acting freely on $N - F$ where F is a codimension 2 submanifold fixed by each $g \in G$. N is called a cyclic cover of N/G ramified along F. Then Sign $N - d$ Sign N/G depends only on F and its normal bundle in N. In [3] Hirzebruch gives an expression for this difference as a formal series in Sign $F^{(r)}$ with rational functions of d as coefficients, where $F^{(r)}$ denotes the r-fold self-intersection of F in N. Working modulo powers of 2 we give explicit formulas for the coefficients.

THEOREM 2. *Let N be a d-fold cyclic cover ramified over F.*

Then for $d \equiv 2 \bmod 4$,

$$\text{Sign } N - d \text{ Sign } N/d \equiv -\text{Sign } F \cdot F \bmod 32. \tag{6}$$

For $d \equiv 0 \bmod 4$,

$$\begin{aligned}\text{Sign } N - d \text{ Sign } N/G \equiv & -5 \text{ Sign } F \cdot F + 4 \text{ Sign } F^{(4)} \\ & -4 \text{ Sign } F^{(6)} + - \cdots \bmod 16.\end{aligned} \tag{7}$$

For d odd

$$\begin{aligned}\text{Sign } N - d \text{ Sign } N/G &\equiv (d^2-1)(-3 \text{ Sign } F^{(2)} + \text{Sign } F^{(4)} \\ &\quad -3 \text{ Sign } F^{(6)} + \text{Sign } F^{(8)} - + \cdots) \bmod 64 \\ &\equiv (d^2-1)(\text{Sign } F^{(2)} + \text{Sign } F^{(4)} + \cdots) \bmod 32 \\ &\equiv 0 \bmod 8.\end{aligned} \tag{8}$$

COROLLARY 5. *If N^{2n} has an almost complex structure preserved by the action of $G = Z/d$, then for d odd*

$$\text{Sign } N - d \text{ Sign } N/G \equiv 8\varepsilon(d) c_{n-1}(N) \cap i_*[F] \bmod 16.$$

Here c denotes the Chern class and $i\colon F \hookrightarrow N$ is the inclusion.

4. Proof of Theorem 1

Let

$$\phi_l(z) = \frac{1}{lz} \frac{(1+z)^l - (1-z)^l}{(1+z)^l + (1-z)^l}.$$

Then

$$\sum_{m=0}^{\infty} \text{Sign } X_{2m}(d) z^{2m} = \frac{d}{1-z^2} \prod_{i=1}^{r} \phi_{d_i}(z)$$

where $\underline{d} = (d_1, \ldots, d_r)$. This is Hirzebruch's formula [**2**, 22.1.1] with $y = 1$ and $k = 0$. Now $\phi_l(z)$ can be expanded as a power series in z^2 whose coefficients are polynomials in l with rational coefficients. In fact these coefficients have only odd denominators, that is $\phi_l(z) \in Z_{(2)}[l][[z^2]]$ where

$$Z_{(2)} = \{a/b\colon\ a, b \in Z \text{ and } b \text{ is odd}\}.$$

To see this observe that

$$\phi_l(z) = \frac{1}{lz} \tanh(l \operatorname{artanh} z)$$

and that the power series for $\tanh x$ and $\operatorname{artanh} x$ have only odd powers of x and coefficients with odd denominators.

Next note that $\phi_l(z)$ is an even function of l, so the coefficients are polynomials in l^2, and that $\phi_l(z) - 1$ vanishes for $l = \pm 1$. Therefore $\phi_l(z) - 1$ is divisible by $l^2 - 1$ in $Z_{(2)}[l^2][[z^2]]$. Moreover for l odd, $l^2 \equiv 1 \bmod 8$, and hence $(\phi_l(z) - 1)/(l^2 - 1) \bmod 8$ is independent of l. Taking $l = 3$, we find this value is $-\frac{1}{3}z^2/(1 + 3z^2)$. Since if l is odd $\frac{1}{3}(l^2 - 1) \equiv 3(l^2 - 1) \bmod 64$,

$$l \text{ odd} \Rightarrow \phi_l(z) \equiv 1 - (l^2 - 1)\frac{3z^2}{1 + 3z^2} \bmod 64. \tag{9}$$

Similarly if $l \equiv 2 \bmod 4$, then $l^2 \equiv 4 \bmod 32$, so

$$l \equiv 2 \bmod 4 \Rightarrow \phi_l(z) \equiv \frac{1}{1 + z^2} \bmod 32. \tag{10}$$

Finally if $l \equiv 0 \bmod 4$, then $l^2 \equiv 0 \bmod 16$, so

$$\phi_l(z) \equiv \phi_4(z) = \frac{1 + z^2}{1 + 6z^2 + z^4} \bmod 16.$$

Now

$$\frac{(1 + z)^2}{1 + 6z^2 + z^4} = 1 - \frac{4z^2}{1 + 6z^2 + z^4} \equiv 1 - \frac{4z^2}{(1 + z^2)^2} \bmod 16,$$

so

$$l \equiv 0 \bmod \Rightarrow \phi_l(z) \equiv \frac{1}{1 + z^2}\left(1 - \frac{4z^2}{(1 + z^2)^2}\right) \bmod 16. \tag{11}$$

We next observe that (9) implies the congruence

$$\prod_{d_i \text{ odd}} \phi_{d_i}(z) \equiv \prod \left(1 - (d_i^2 - 1)\frac{3z^2}{1 + 3z^2}\right)$$
$$\equiv 1 - \sum_{d_i \text{ odd}} (d_i^2 - 1)\frac{3z^2}{1 + 3z^2} \bmod 64$$

because each $d_i^2 - 1$ is divisible by 8.

Moreover the function $d \mapsto d^2 - 1 \bmod 64$ from the multiplicative semigroup of odd integers to the additive group $8Z/64Z$ is a homomorphism since, if k and l are odd,

$$\begin{aligned} k^2l^2 - 1 &= (k^2-1)+(l^2-1)+(k^2-1)(l^2-1) \\ &\equiv (k^2-1)+(l^2-1) \bmod 64. \end{aligned}$$

Hence

$$\prod_{d_i \text{ odd}} \phi_{d_i}(z) \equiv 1-(D^2-1)\frac{3z^2}{1+3z^2} \bmod 64 \tag{12}$$

where $D = d_{s+1} \cdots d_r$ is the product of the odd entries in the multidegree.

Combining (12) with (10) and (11) and observing that 2^{s+s^*} divides d we obtain

$$\begin{aligned} &\sum_{m=0}^{\infty} \operatorname{Sign} X_{2m}(\underline{d}) z^{2m} \\ &\equiv \frac{d}{1-z^2}\left(1-(D^2-1)\frac{3z^2}{1+3z^2}\right)\left(\frac{1}{1+z^2}\right)^s\left(1-\frac{4z^2}{(1+z^2)^2}\right)^{s^*} \\ &\equiv \frac{d}{(1-z^2)(1+z^2)^s}\left(1-(D^2-1)\frac{3z^2}{1+3z^2}-\frac{4s^*z}{(1+z^2)^2}\right) \\ &\qquad \bmod 2^{\max\{6,5+s,4+s+s^*\}} \end{aligned} \tag{13}$$

This is equivalent to (3) if we define the α's, β's, and γ's as the coefficients of the following expansions:

$$\begin{aligned} \sum_{m=0}^{\infty} \alpha_{m,s} z^{2m} &= \frac{1}{1-z^2}\frac{1}{(1+z^2)^s} \\ \sum_{m=0}^{\infty} \beta_{m,s} z^{2m} &= \frac{1}{1-z^2}\frac{1}{(1+z^2)^s}\frac{3z^2}{1+3z^2} \\ \sum_{m=0}^{\infty} \gamma_{m,s} z^{2m} &= \frac{1}{1-z^2}\frac{z^2}{(1+z^2)^{s+2}}. \end{aligned} \tag{14}$$

Note $\gamma_{m,s} = \alpha_{m-1,s+2}$. For our purpose the α's are computed mod 64, the β's mod 8, and the γ's mod 4.

5. Proofs of the corollaries

For complete intersections of odd degree we have $s = 0$. From (14) $\alpha_{m,0} = 1$ for all m. Since $s^* = 0$, the term in (3) with s^* vanishes. Therefore we need only compute $\beta_{m,0} \bmod 8$. It is easy to check $-\beta_{m,0} \equiv m - 4[m+1/2] \bmod 8$.

Next consider congruences mod 32. If $d \equiv 0 \bmod 8$, the terms in (3)

involving D and s^* vanish mod 32, so the signature is congruent to $d\alpha_{m,s}$ mod 32, If $d \equiv 4$ mod 8, then the term involving D vanishes mod 32, so the signature mod 32 depends only on m, d, s, and s^*. But s^* itself is determined by s since $d \equiv 4$ mod 8 implies $s = 1$, $s^* = 1$ or $s = 2$, $s^* = 0$. If $d \equiv 2$ mod 4, then $s^* = 0$ and we have

$$\text{Sign } X_{2m}(d) \equiv d(\alpha_{m,s} - (D^2 - 1)\beta_{m,s}) \bmod 32.$$

Actual dependence on D can be seen in the examples $X_2(6, 1)$ and $X_2(2, 3)$ of degree 6 in CP_4: Sign $X_2(6, 1) = -64$, Sign $X_2(2, 3) = -16$ and the difference is 16 mod 32.

For the computation of the table mod 16, formula (5) follows from (13) since (i) if $s^* \neq 0$ then $4d \equiv 0$ mod 16 and (ii) $D^2 - 1 \equiv 8\varepsilon(D)$ mod 16. The results of the table follow easily.

6. Proof of theorem 2

Denote the quotient manifold N/G by M and let i: $F \hookrightarrow M$ be the inclusion of the branch set. Then $i_*[F] = dx \cap [M]$ for a class $x \in H^2(M; Z)$. The signatures of M and N are given by

$$\text{Sign } M = \{L(M)\}[M]$$
$$\text{Sign } N = d\{\phi_d\ (\tanh x)L(M)\}[M]$$

where $L(M)$ is the Hirzebruch L-genus, see [**3**] or [**9**, (8.3)].

Let $t = \tanh i^*x \in H^*(F; Q)$. $F^{(r)}$ is the intersection of r slightly deformed copies of F which meet transversely. The signatures of these self-intersections are given by

$$\text{Sign } F^{(r+1)} = \{t^r L(F)\}[F], \qquad r \geq 0.$$

Now

$$\begin{aligned}\text{Sign } N - d \text{ Sign } M &= d\{(\phi_d(\tanh x) - 1)L(M)\}[M] \\ &= d\left\{\frac{\phi_d(\tanh i^*x) - 1}{\tanh i^*dx} L(F)\right\}[F] \\ &= d\left\{\frac{\phi_d(t) - 1}{t\phi_d(t)} L(F)\right\}[F]\end{aligned}$$

If $d \equiv 2$ mod 4, then by (10) $\phi_d(t) = 1/(1 + t^2)$ mod 32 and hence $(\phi_d(t) - 1)/d\phi_d(t) = -t$. This proves (6).

For $d = 2$ the congruence can be replaced by equality. If $d \equiv 0$ mod 4 then $\phi_d(t) \equiv \phi_4(t)$ mod 16 and

$$\frac{\phi_d(t) - 1}{t\phi_d(t)} \equiv -t\frac{5 + t^2}{1 + t^2} \equiv t(-5 + 4t^2 - 4t^4 + - \cdots) \bmod 16,$$

from which (7) follows. If d is odd, $\phi_d(t)-1 \doteq -(d^2-1)3t^2/(1+3t^2)$ mod 64 by (9) and also $1/\phi_d(t) \equiv 1 \bmod 8$, so

$$\frac{\phi_d(t)-1}{t\phi_d(t)} \equiv -(d^2-1)\frac{3t}{1+3t^2} \bmod 64$$
$$\equiv (d^2-1)t(-3+t^2-3t^4+t^6-+\cdots) \bmod 64.$$

From this (8) follows.

To prove Corollary 5 we must show

$$\sum_{r=1}^{\infty} \operatorname{Sign} F^{(2r)} \equiv c_{n-1}(N) \cap i_*[F] \bmod 2. \tag{15}$$

Now $\operatorname{Sign} F^{(r)}$ is 0 for r odd and is congruent mod 2 to the Euler characteristic $e(F^{(r)})$ in any case. Denote by $i^{(r)}$ the inclusion of $F^{(r)}$ in N. If F is dual to $y \in H^2(N; Z)$, then $F^{(r)}$ is dual to y^r, that is $i_*^{(r)}[F^{(r)}] = y^r \cap [N]$. The normal bundle of $i^{(r)}$ is the sum of r complex line bundles with Chern class $1+i^{(r)*}y$. Hence

$$e(F^{(r)}) = \langle c(F^{(r)}), [F^{(r)}]\rangle = \langle i^{(r)*}c(N)(1+i^{(r)*}y)^{-r}, [F^{(r)}]\rangle$$
$$= \langle c(N)(1+y)^{-r}, i_*^{(r)}[F^{(r)}]\rangle = \langle c(N)(1+y)^{-r}y^r, [N]\rangle.$$

Now

$$\sum_{r=1}^{\infty} (1+y)^{-r}y^r = y,$$

so

$$\sum_{r=1}^{\infty} e(F^{(r)}) = c_{n-1}(N)y \cap [N] = c_{n-1}(N) \cap i_*[F].$$

This shows (15) and completes the proof of Corollary 5.

7. Behavior of Rokhlin's invariant in cyclic covers

First we recall Rokhlin's definitions. Let V be a 4-manifold with $H_1(V; Z/2) = 0$ and let K be an oriented characteristic surface, that is a 2-dimensional submanifold whose homology class mod 2 in V is dual to $w_2(V)$. Let q: $H_1(K; Z/2) \to Z/2$ be the quadratic function defined as follows: given a class $\alpha \in H_1(K; Z/2)$ there is a surface S with boundary such that $\partial S \subset K$ represents α and K meets S transversely at j interior points. Let i be the obstruction to extending across S the field of unit normal vectors to ∂S in K. Then $q(\alpha) = i + j \bmod 2$ is a quadratic function associated to the intersection pairing on K and Rokhlin's invariant $k(V, K)$ is the Arf invariant of q.

Rokhlin's formula [7] is

$$\text{Sign } V \equiv K \cdot K + 8k(V, K) \bmod 16. \tag{16}$$

It follows in particular that $k(V, K)$ depends only on the integer homology class of K.

The behavior of Rokhlin's invariant in ramified covers is given by the following

PROPOSITION. *Let π: $V' \to V$ be a cyclic cover of odd order d ramified over F. Let K' be an invariant surface in V' and K its quotient. Then K' is characteristic for V' iff K is for V and*

$$k(V', K') = k(V, K) + \varepsilon(d) K \cdot F \in Z/2. \tag{17}$$

Proof. If X is any surface in V let tX, the transfer, be its inverse image in V'. The basic formula for intersections is

$$tX \cdot tY = dX \cdot Y \tag{18}$$

since we may assume $X \cap Y$ is disjoint from F. Since K' is invariant, $K' = tK$. If X' is a surface in V', then as homology classes $t\pi X' = dX'$. Now suppose K is characteristic. Then $K' \cdot X' = tK \cdot X' \equiv tK \cdot dX' = tK \cdot t\pi X' = dK \cdot \pi X' \equiv d\pi X' \cdot \pi X' = t\pi X' \cdot t\pi X' = d^2 X' \cdot X' \equiv X' \cdot X'$, so K' is characteristic. The converse is similar, but shorter.

Formula (17) is a consequence of (16) and (8). Modulo 2 we have

$$k(V', k') \equiv \tfrac{1}{8}(\text{Sign } V' - K' \cdot K')$$

by (16)

$$\equiv \tfrac{1}{8}(d \text{ Sign } V + (d^2 - 1)F \cdot F - dK \cdot K)$$

by (8) and (17)

$$\equiv d\tfrac{1}{8}(\text{Sign } V - K \cdot K) + \varepsilon(d) F \cdot F$$
$$\equiv k(V, K) + \varepsilon(d) K \cdot F$$

since K is characteristic.

There is also a simple geometric proof of (17) which does not depend ultimately on the G-signature theorem. The restriction of π to K', π: $K' \to K$, is a d-fold cover branched over $K \cap F$ and transfer induces an injection t: $H_1(K) \hookrightarrow H_1(K')$ on homology with $Z/2$ coefficients. An $\alpha \in H_1(K)$ can be represented by $\partial S \subset K$ where $S \cap K \cap F = \phi$. Then $t\alpha$ is represented by $\partial(tS)$. Now tS meets $K' = tK$ in $j' = dS \cdot K$ points. Lifting to tS a vector field with isolated, nondegenerate zeros normal to S we see $i' = di$ and hence $q'(t, x) = q(\alpha) \in Z/2$. Hence $q' \mid \text{image } t = q$. Also q' is invariant under the covering transformations (which act on all of V').

Thus we can apply [**10**, Theorem 4] which, written additively, gives

$\operatorname{Arf} q' = \operatorname{Arf} q + \varepsilon(d) K \cdot F$ since $e(K)$ is even and the Jacobi symbol $(2 \mid d) = (-1)^{\varepsilon(d)}$. The proof in [**10**] is a simple counting argument.

To deduce formula (2) note first that, since d is odd, $X_1(\underline{d})$ is characteristic for $X_2(\underline{d})$ and $X_1(\underline{d}) \cdot X_1(\underline{d}) = d$ so (2) follows from Rokhlin's formula (16) if we show that $k(X_2(\underline{d}), X_1(\underline{d})) = \varepsilon(d)$. This is proved by induction on r. For $r = 0$ we have $d = 1$ and $k(CP_2, CP_1) = 1$ by (15). Let $\underline{d} = (d_1, \ldots, d_r)$ and $\underline{e} = (d_2, \ldots, d_r)$ where $d_1 \leq d_j$ for $1 \leq j \leq r$. By [**11**, (2.1)] there is a regular branched cyclic cover $X_2(\underline{d}) \to X_2(\underline{e})$ branched over $F = X_1(\underline{d})$. Referring to the proof in [**11**, § 2] we see that the hyperplane section K' of $X_2(\underline{d})$ obtained by setting the last variable equal to 0 is a characteristic surface invariant under the group action; K' and F are homologous in $X_2(\underline{d})$ but not equal. The quotient K is the hyperplane section $X_1(\underline{e})$ of $X_2(\underline{e})$ and $K \cap F = X_0(\underline{d})$ so $K \circ F = d \equiv 1 \bmod 2$. Hence by (17)

$$k(X_2(\underline{d}), X_1(\underline{d})) = k(X_2(\underline{e}), X_1(\underline{e})) + \varepsilon(d_1).$$

Therefore

$$k(X_2(\underline{d}), X_1(\underline{d})) = \varepsilon(d_1) + \cdots + \varepsilon(d_r) = \varepsilon(d).$$

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