MPIM, Specialized workshop

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Geometric quantization and topological recursion

November 24-28th, 2014

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Organizers : Gaëtan Borot, Peter Teichner, Don Zagier.

Two 4h mini-courses will be held at this occasion :

The Hitchin connection and the Witten-Reshetikhin-Turaev TQFT

Jørgen Ellegaard Andersen (QGM Aarhus)

Topological recursion and moduli spaces

Bertrand Eynard (IPHT CEA Saclay & CRM Montréal)

The courses will take place in MPIM Lecture room, 9h-11h, Vivatsgasse 7.

Scope. Informally, geometric quantization is a way to associate a Hilbert space of "states" to a moduli space $\mathcal{M}$ of classical solutions of a quantum field theory. Very often, one has to choose an extra structure (like a complex structure) on $\mathcal{M}$ in this construction, and show that the resulting (projective) Hilbert space does not depend on this choice up to isomorphism. When it exists, the Hitchin connection enables this program. These ideas have been fruitful in the context of Chern-Simons theory in 3-manifolds and give rise to TQFTs. The question of asymptotics of knot states when $\hbar \to 0$ has been intensively studied for 15 years, with up to now only very partial results (valid at leading order for certain knots, or at all orders for very simple knots). A famous motivation for this study is the volume conjecture of Kashaev, stating that the sequence of colored Jones polynomials grows with a rate related to the hyperbolic volume of the knot complement, and generalizations thereof. The problem therefore ties together the algebraic and combinatorial aspects of knot theory, with the geometry of character varieties and underlying TQFTs.

The topological recursion is a general structure behind the space of solutions of Virasoro constraints. The initial data of the recursion is mainly the choice of a lagrangian $\mathcal{L}$ in $\mathbb{C}^2$, the restriction to $\mathcal{L}$ of a primitive of the canonical symplectic form on $\mathbb{C}^2$, and a certain collection of sections of bundles over $\mathcal{L}$. The topological recursion has many structural properties, in particular under deformation of the initial data and change of primitive of the symplectic form. For various choices of initial data, it computes amplitudes in topological strings, $2d$ quantum gravity, intersection numbers on $\overline{M}_{g,n}$, etc. It was conjectured – and checked numerically on some examples – that the recursion can be used to compute the all-order asymptotic expansion of the colored Jones polynomial of a knot $K \subseteq \mathbb{S}_3$, by considering as lagrangian the $\mathrm{SL}_2(\mathbb{C})$ character variety of $\mathbb{S}_3 \setminus K$ and initial data depending on its geometry.