HW 9. Please return Dec. 18

① Set $\Omega_n(X) := \left\{ (M, f) \;\middle|\; \begin{array}{l} M^n \text{ closed orientable} \\ \text{mfd., } f: M \to X \text{ cont.} \end{array} \right\} \Big/ \sim$

where an equivalence $\sim$ is given by a compact oriented $(n+1)$-mfd W and $F: W \to X$: $(\partial_+ W, F/_{\partial_0}) \sim (\partial_- W, F/_{\partial_1})$ whenever $\partial W = (-\partial_0) \amalg (\partial_1)$.

(a) Show that the Hurewicz map factors as $h_n: \pi_n(X, x_0) \xrightarrow{a_n} \Omega_n(X) \xrightarrow{b_n} H_n(X)$ and that b_1 and b_2 are iso.s for CW-complexes X.

(b) Derive the 5-term exact sequence

$$H_2(G) \to H_2(Q) \to N/[G,N] \to H_1(G) \to H_1(Q) \to 0$$

associated to a group extension $N \rightarrowtail G \twoheadrightarrow Q$

② Consider a group homom. $\varphi: A \to B$ between nilpotent groups. Show

(a) φ is onto $\iff$ $H_1(\varphi)$ is onto

(b) φ is an iso. $\iff$ $H_1(\varphi)$ is an iso. and $H_2(\varphi)$ is onto

Stallings' Theorem