HW8 Please submit Dec. 4

① Consider $f = (f_0, f_3) : (D^2 \amalg D^2, \partial) \hookrightarrow (M^4, \partial)$

Define $\mathrm{Bing}(f) = (f_1, f_2, f_3) : (\amalg^3 D^2, \partial) \hookrightarrow$

such that $f_0 = W_{12}$ pairs the two intersections between f_1 and f_2 and such that $\partial(f_1, f_2)$ is the Bing double of $\partial f_0 : S^1 \hookrightarrow \partial M$.

a) Compute $\lambda_3(\mathrm{Bing}(f))$ in terms of $\lambda_2(f)$.

b) Extend your definition & computation to $f : S^2 \amalg S^2 \hookrightarrow M^4$.

② Let M be a connected 4-mfd. with $\pi := \mathrm{image}\left(\pi_1(\partial M, m_0) \to \pi_1(M, m_0)\right)$

a) For any $\lambda \in \mathbb{Z}\pi$, there is an $f : (D^2 \amalg D^2, \partial) \hookrightarrow (M, \partial)$ with $\lambda_2(f) = \lambda$.

b) For any $\lambda \in \mathbb{Z}\pi^2$ there is a non-repeating Whitney tower of order 1 $\mathcal{W} = (f_1, f_2, f_3 ; W_{12})$ with $f_i : (D^2, \partial) \hookrightarrow (M, \partial)$ such that $\lambda_3(\mathcal{W}) = \lambda$.