

HW 4, please submit on Nov. 6

① (a) Describe geometrically an abelian group structure on  $\left\{ (f, w) \mid \begin{array}{l} S^2 \xrightarrow{f} M \text{ generic} \\ w \text{ whisker } m_0 \text{ to } f(s_0) \end{array} \right\} / \text{regular homotopy}$

s.t. the projection to  $\pi_2(M, m_0)$  is an epimorphism with infinite cyclic kernel.

(b) Does this group extension split?

(c) Show  $\mu(f_1 + f_2) = \mu(f_1) + \mu(f_2) + [\chi(f_1, f_2)]$

② (a) Draw a non-trivial knot which is slice. Same for a 2-component link (whose components are not separated by a plane).

(b) Show that the Borromean rings are not slice. Hint: Either reduce to 2-components or consider

$$\mathcal{D} := \left\{ \Delta : \mathbb{D}^2 \amalg \mathbb{D}^2 \amalg \mathbb{D}^2 \hookrightarrow \mathbb{D}^4 \mid \begin{array}{l} \Delta_1 \text{ is an embedding,} \\ \text{disjoint from } \Delta_2, \Delta_3 \end{array} \right\}$$

$\partial \downarrow$

$$\mathcal{L} := \left\{ L : S^1 \amalg S^1 \amalg S^1 \hookrightarrow S^3 \mid \text{lk}(L_i, L_j) = 0 \quad i=2,3 \right\}$$