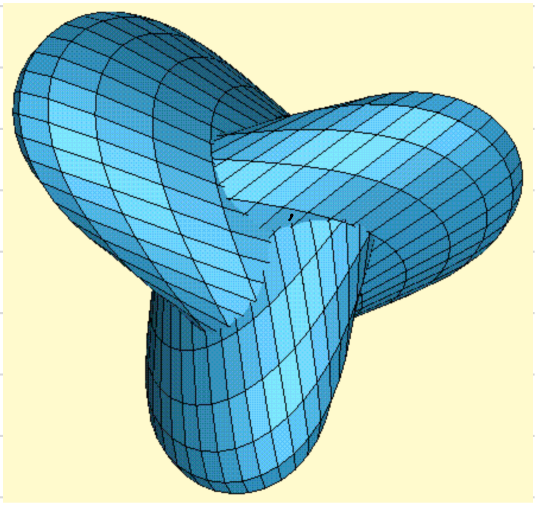
HW 2, due Oct. 23

(1) Boy's surface:



Draw a sequence of level curves for an immersion $\mathbb{RP}^2 \hookrightarrow \mathbb{R}^3$ which has a 3-fold symmetry.

Hint: One can use one minimum & 3 maxima.

(2) Let $f: \mathbb{RP}^2 \to \mathbb{R}^3$ be a stable map, i.e. its level curves are minima, saddles, maxima and type I, II, III moves.

Show that #min. − #saddles + #max. = 1 and that there is an even number of type I moves.

(3) Show that every smooth embedding $S^2 \hookrightarrow \mathbb{R}^3$ is isotopic to the round sphere $S^2 \subset \mathbb{R}^3$.