

$p = 5$

$k$	$d_k^+$	$d_k^-$
2	0	0
4	1	0
6	0	1
8	2	1
10	1	2
12	3	2
14	2	3
16	4	3
18	3	4
20	5	4
22	4	5
24	6	5
26	5	6

$p = 23$

$k$	$d_k^+$	$d_k^-$
2	0	2
4	4	1
6	3	6
8	8	5
10	7	10
12	12	9
14	11	14
16	16	13
18	15	18
20	20	17
22	19	22
24	24	21
26	23	26

$p = 101$

$k$	$d_k^+$	$d_k^-$
2	1	7
4	16	9
6	17	24
8	33	26
10	34	41
12	50	43
14	51	58
16	67	60
18	68	75
20	84	77
22	85	92
24	101	94
26	102	109

$$d_k^\pm = \dim S_k(p)^\pm$$

$p = 5$

$k$	$d_k^+$	$d_k^-$
2	0	0
4	1	0
6	0	1
8	2	1
10	1	2
12	3	2
14	2	3
16	4	3
18	3	4
20	5	4
22	4	5
24	6	5
26	5	6

$$\Delta_k = \pm 1$$

$p = 23$

$k$	$d_k^+$	$d_k^-$
2	0	2
4	4	1
6	3	6
8	8	5
10	7	10
12	12	9
14	11	14
16	16	13
18	15	18
20	20	17
22	19	22
24	24	21
26	23	26

$$\Delta_k = \pm 3$$

$p = 101$

$k$	$d_k^+$	$d_k^-$
2	1	7
4	16	9
6	17	24
8	33	26
10	34	41
12	50	43
14	51	58
16	67	60
18	68	75
20	84	77
22	85	92
24	101	94
26	102	109

$$\Delta_k = \pm 7$$

$$d_k^\pm = \dim S_k(p)^\pm$$

$$\Delta_k = d_k^+ - d_k^-$$

$$p = 5, N = 23$$

Table of dimension splits with  $(d_{k,g}^+, d_{k,g}^-)$  in the  $k$ - $g$  entry.

$k \setminus g$	$f$	$f[1]$	$f[2]$	$f[3]$
2	(3, 2)	—	(0, 0)	—
4	—	(2, 3)	—	(0, 0)
6	(5, 5)	—	(3, 2)	—
8	—	(5, 5)	—	(2, 3)
10	(8, 7)	—	(5, 5)	—
12	—	(7, 8)	—	(5, 5)
14	(10, 10)	—	(8, 7)	—
16	—	(10, 10)	—	(7, 8)
18	(13, 12)	—	(10, 10)	—
20	—	(12, 13)	—	(10, 10)
22	(15, 15)	—	(13, 12)	—
24	—	(15, 15)	—	(12, 13)

Here  $f = q + 2q^2 + 2q^4 + 4q^5 + q^7 + \dots$  in  $S_2(23, \mathbb{F}_5)$ .

$$p = 5, N = 23$$

Table of dimension splits with  $(d_{k,g}^+, d_{k,g}^-)$  in the  $k$ - $g$  entry.

$k \backslash g$	$f$	$f[1]$	$f[2]$	$f[3]$
2	(3, 2)	—	(0, 0)	—
4	—	(2, 3)	—	(0, 0)
6	(5, 5)	—	(3, 2)	—
8	—	(5, 5)	—	(2, 3)
10	(8, 7)	—	(5, 5)	—
12	—	(7, 8)	—	(5, 5)
14	(10, 10)	—	(8, 7)	—
16	—	(10, 10)	—	(7, 8)
18	(13, 12)	—	(10, 10)	—
20	—	(12, 13)	—	(10, 10)
22	(15, 15)	—	(13, 12)	—
24	—	(15, 15)	—	(12, 13)

Here  $f = q + 2q^2 + 2q^4 + 4q^5 + q^7 + \dots$  in  $S_2(23, \mathbb{F}_5)$ .

$$p = 5, N = 23$$

All 7 Galois orbits of eigensystems; the  $k$ - $f$  entry is  $(d_{k,\tau[\frac{k-2}{2}]}^+, d_{k,\tau[\frac{k-2}{2}]}^-)$ .

$k \setminus \tau$	$e$	$e[2]$	$\sigma$	$\sigma[2]$	$t$	$t[2]$	$s$	$s[2]$	$f, f[2]$ $g, g[2]$ $h, h[2]$	Total
2	(0, 0)	(0, 0)	(3, 2)	(0, 0)	(2, 0)	(0, 0)	(0, 1)	(0, 0)	(0, 0)	(5, 6)
4	(2, 1)	(0, 0)	(2, 3)	(0, 0)	(0, 2)	(0, 0)	(1, 0)	(0, 0)	(1, 1)	(18, 16)
6	(1, 2)	(1, 1)	(3, 2)	(5, 5)	(2, 0)	(2, 2)	(0, 1)	(1, 1)	(1, 1)	(28, 30)
8	(2, 1)	(3, 3)	(2, 3)	(5, 5)	(0, 2)	(2, 2)	(1, 0)	(1, 1)	(2, 2)	(42, 40)
10	(2, 3)	(3, 3)	(8, 7)	(5, 5)	(4, 2)	(2, 2)	(1, 2)	(1, 1)	(2, 2)	(52, 54)
12	(5, 4)	(3, 3)	(7, 8)	(5, 5)	(2, 4)	(2, 2)	(2, 1)	(1, 1)	(3, 3)	(66, 64)
14	(4, 5)	(4, 4)	(8, 7)	(10, 10)	(4, 2)	(4, 4)	(1, 2)	(2, 2)	(3, 3)	(76, 78)
16	(5, 4)	(6, 6)	(7, 8)	(10, 10)	(2, 4)	(4, 4)	(2, 1)	(2, 2)	(4, 4)	(90, 88)
18	(5, 6)	(6, 6)	(13, 12)	(10, 10)	(6, 4)	(4, 4)	(2, 3)	(2, 2)	(4, 4)	(100, 102)
20	(8, 7)	(6, 6)	(12, 13)	(10, 10)	(4, 6)	(4, 4)	(3, 2)	(2, 2)	(5, 5)	(114, 112)

- $e$  is the Eisenstein eigensystem in weight 2:  $e(\ell) = 1 + \ell$
- $s$  is a  $\mathbb{F}_{5^4}$ -Galois orbit of 4 eigensystems;  $h$  is an  $\mathbb{F}_{5^3}$ -orbit of 3 eigensystems
- $\sigma$ : Serre weight 2 (peu ramifié);  $t$  and  $s$ : Serre weight 6 (très ramifié);  $f, g, h$ : Serre weight 4