

To a Coxeter system (W, S) one may associate a simplicial complex $CC(W)$ called the Coxeter complex of W .

Let $n = |S|$ denote the rank of W . Its construction is as follows:

- ▶ colour the n faces of the standard $n - 1$ -simplex by the set S ,
- ▶ take one such simplex for each element $w \in W$,
- ▶ glue the simplex corresponding to w to that corresponding to ws along the wall coloured by s .

For example, consider

$$W = \langle s, t \mid s^2 = t^2 = (st)^3 \rangle = \{e, s, t, st, ts, sts\}.$$

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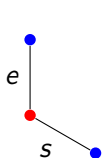


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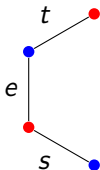
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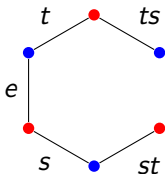
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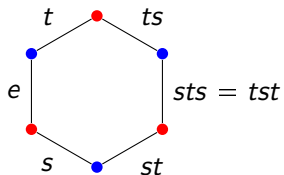
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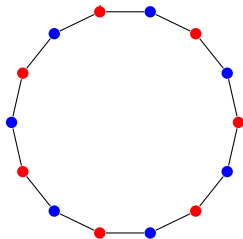
A general dihedral group:

$$W = \langle s, t \mid s^2 = t^2 = (st)^{m_{st}} \rangle.$$

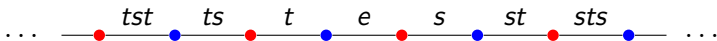
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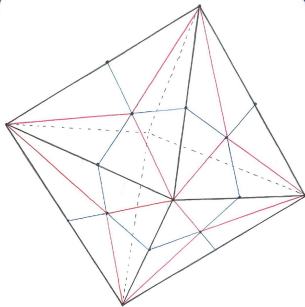
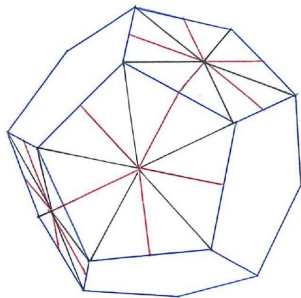
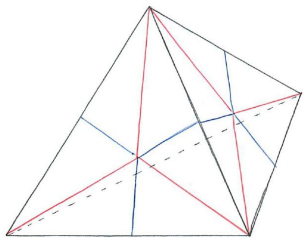


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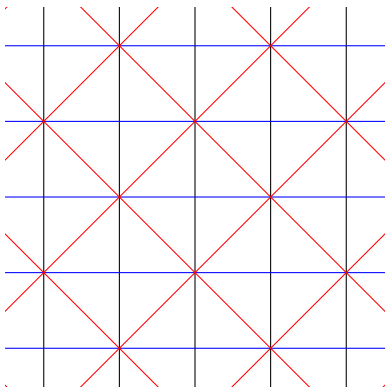


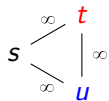
What is the situation in rank 3?

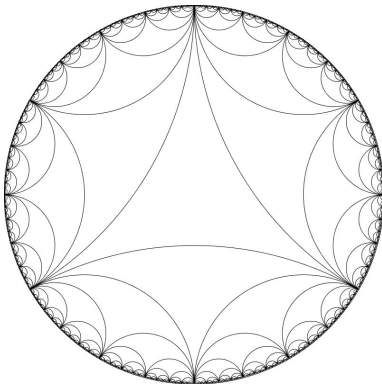
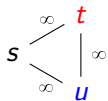
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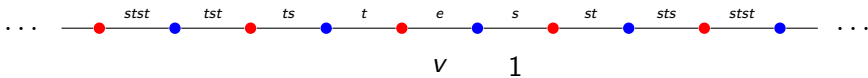
s ⁴ — t ⁴ — u

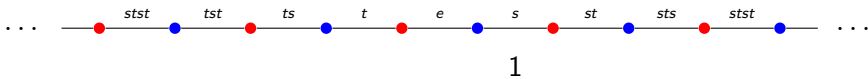




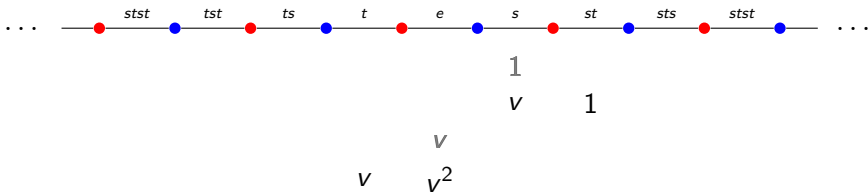


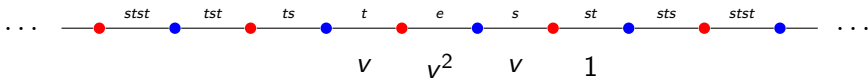
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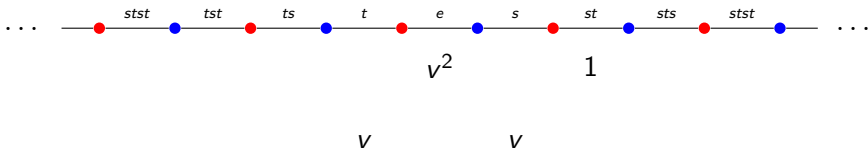


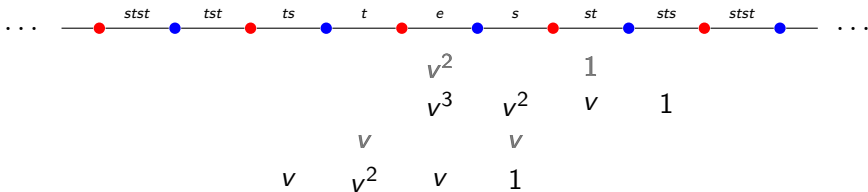


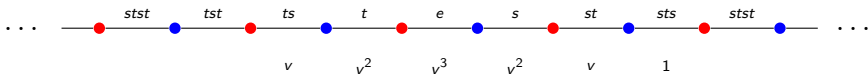
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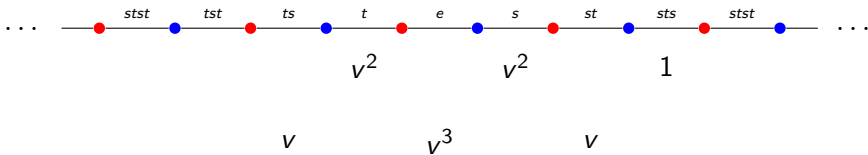


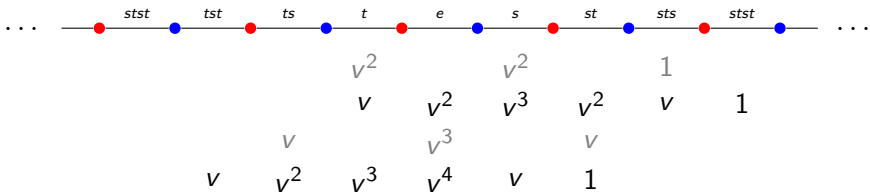


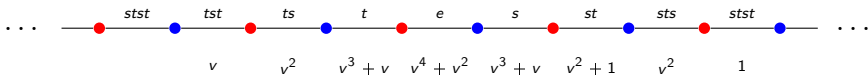


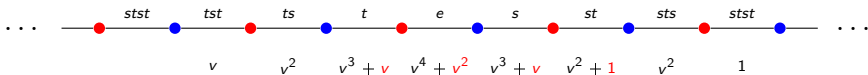


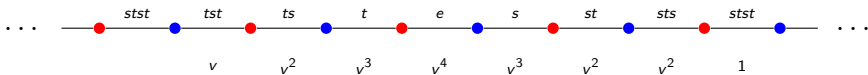


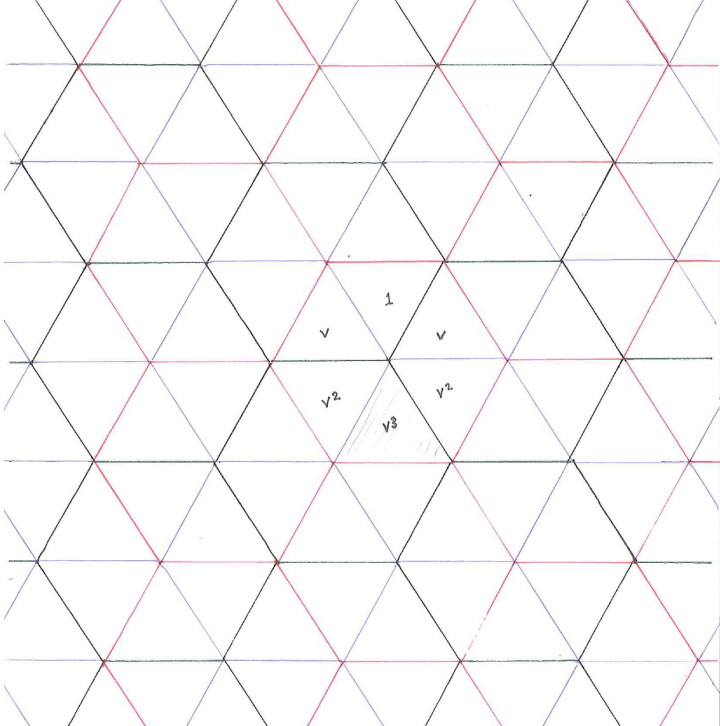


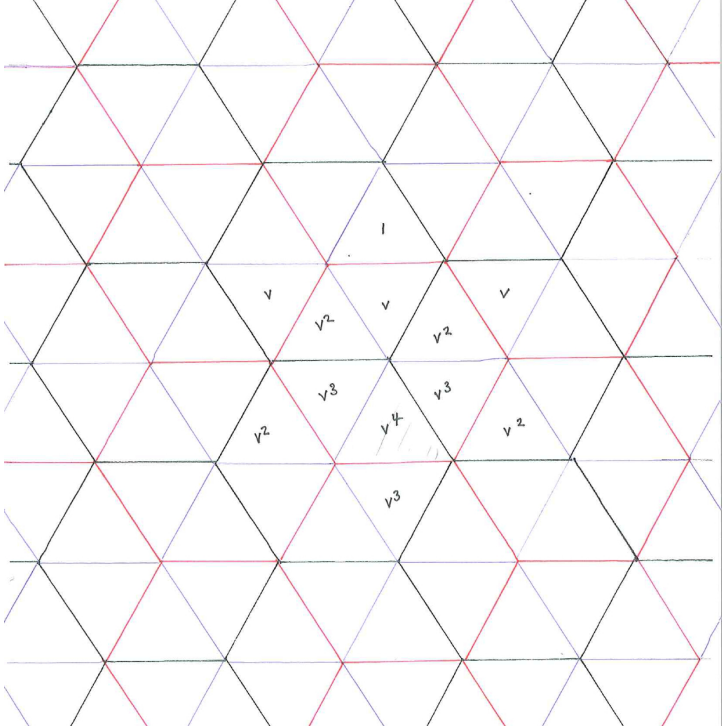


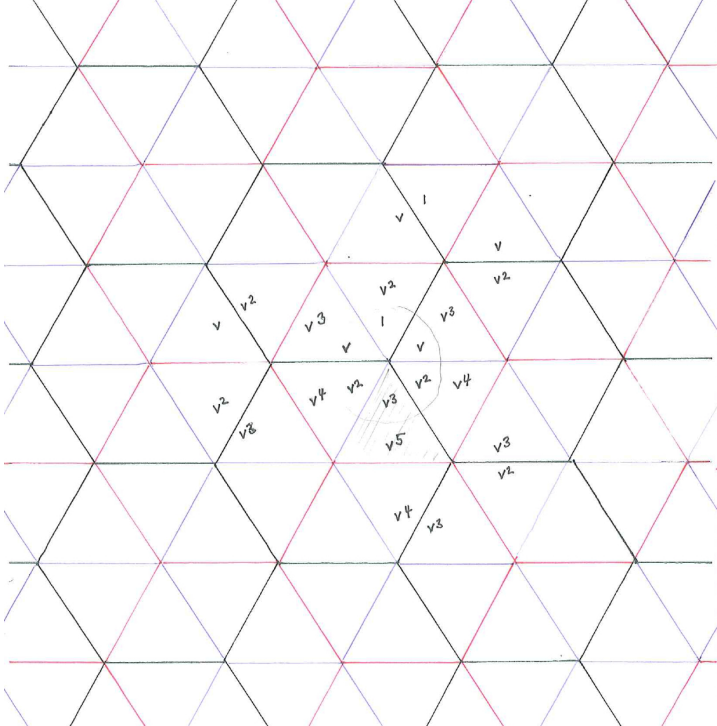


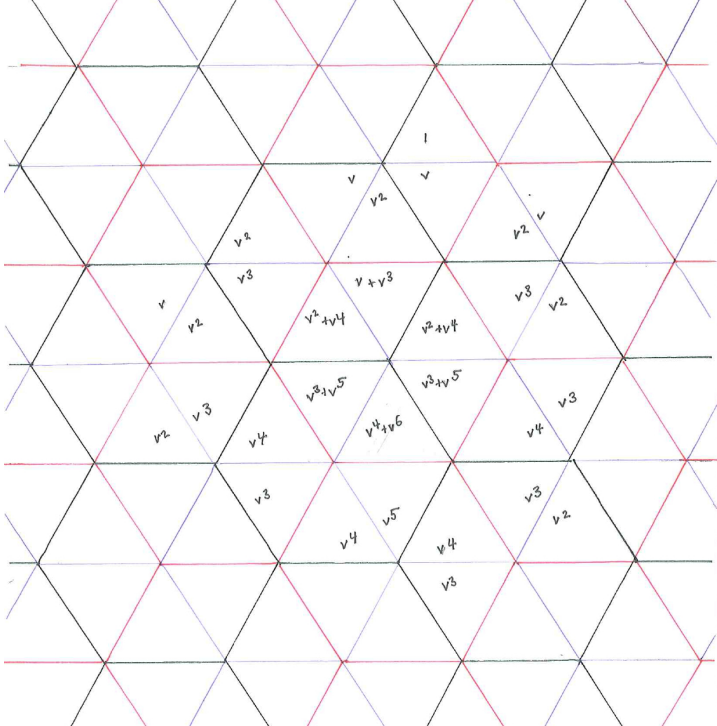


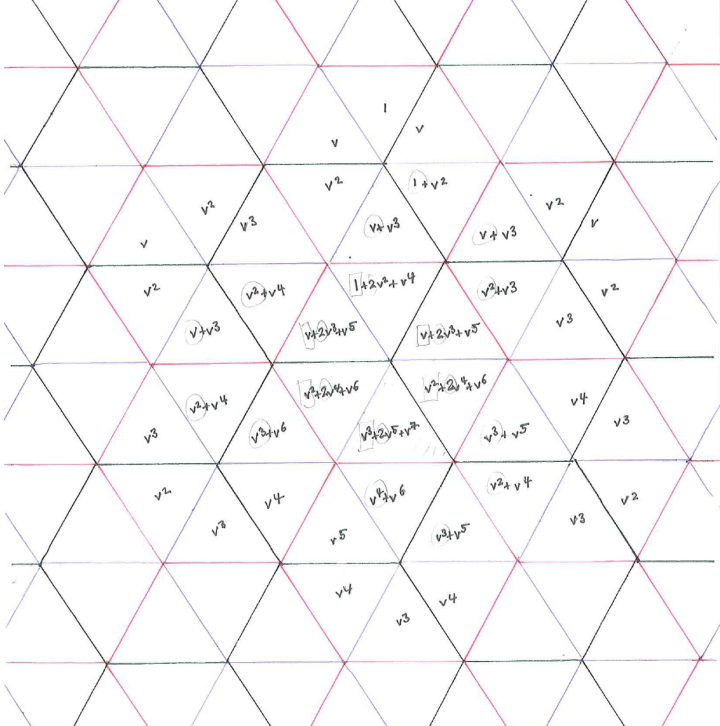


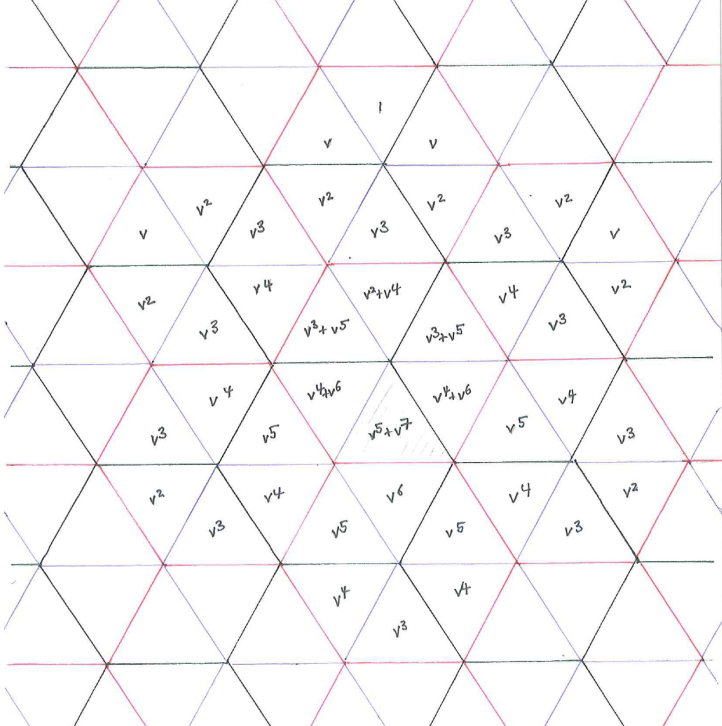


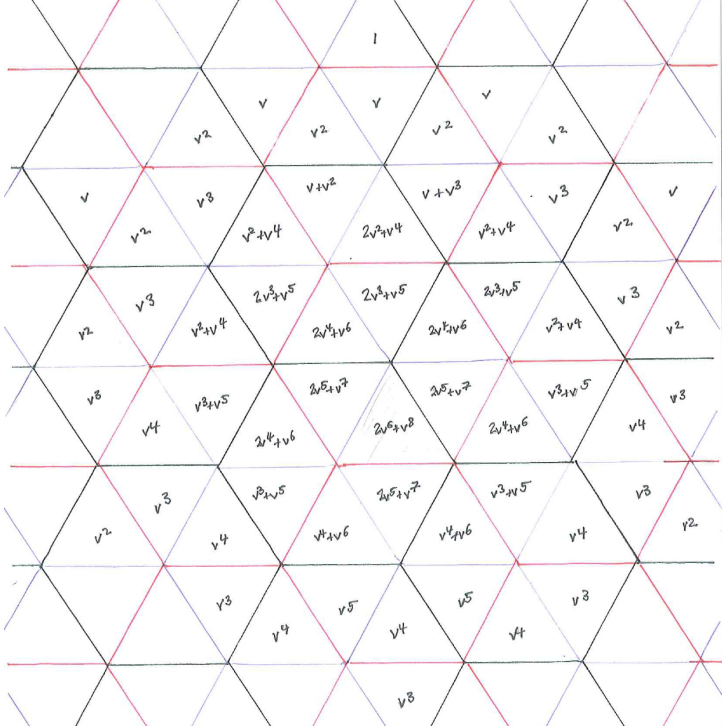


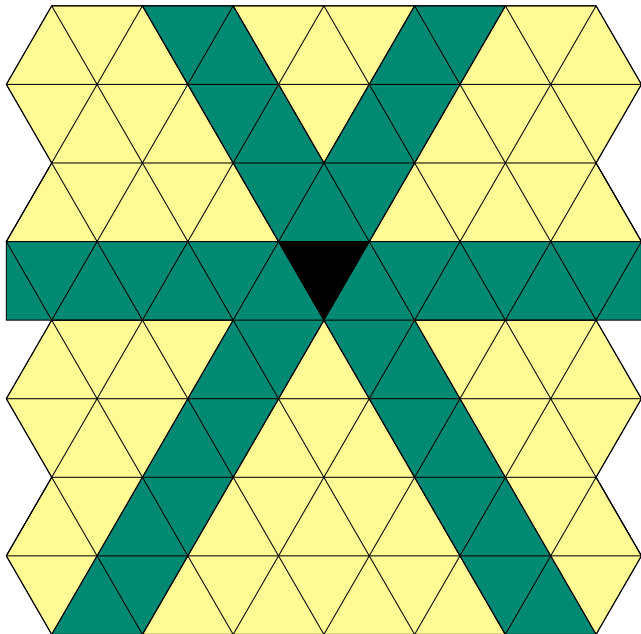


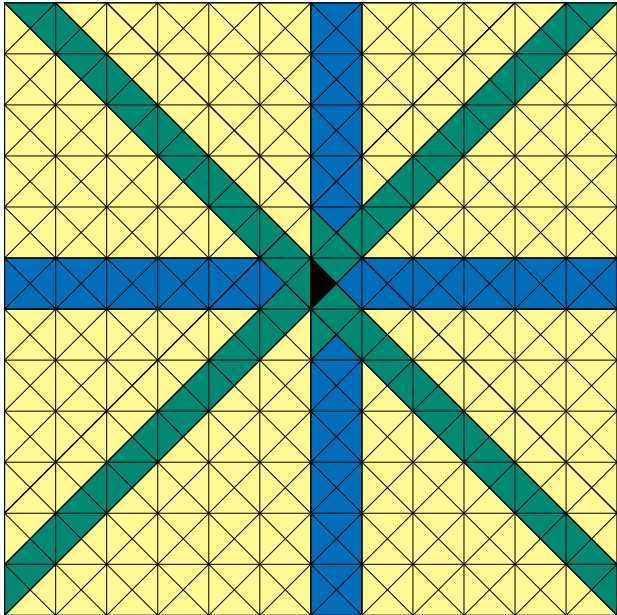


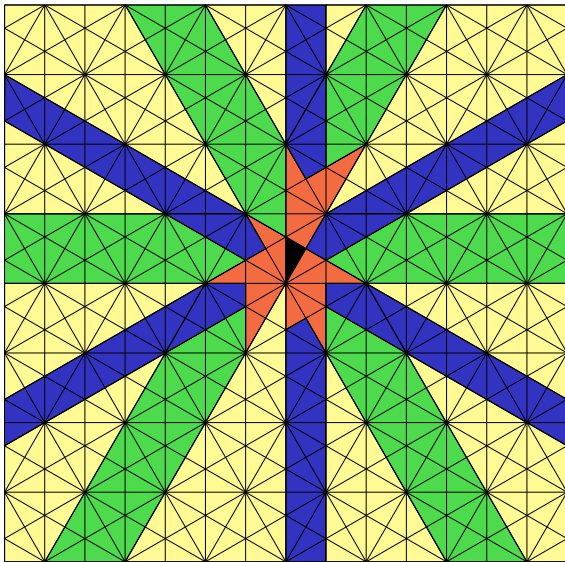




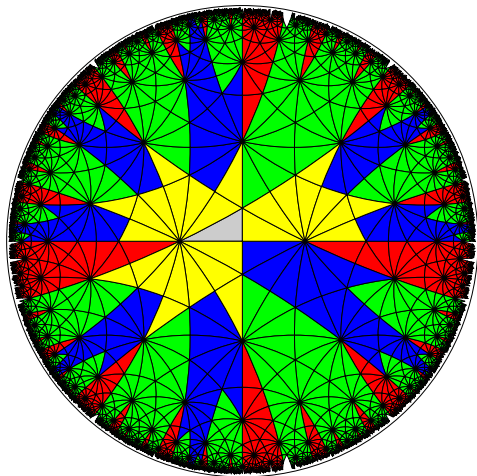


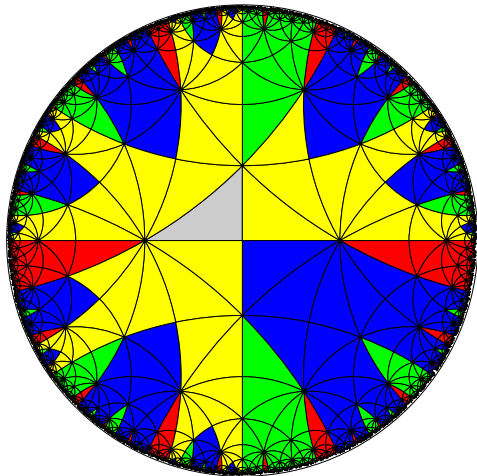






Thanks to Jérémie Guilhot for the images of cells in affine Weyl groups.





For more images of two-sided cells in hyperbolic groups see [Paul Gunnell's web page](#).