SEMINAR NOTES: QUANTIZATION OF HITCHIN'S INTEGRABLE SYSTEM AND HECKE EIGENSHEAVES (SEPT. 8, 2009)

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1. Hecke eigensheaves

The general topic of this seminar can be broadly defined as Geometric Representation Theory with the focus on Geometric Langlands Correspondence.

The latter takes as an input an algebraic curve X (over a fixed base field k assumed algebraically closed and of characteristic 0) and a reductive group G. Let Bun_G denote the algebraic stack of principal G-bundles on X. Our main object of study is the category $\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)$, the category of D-modules on Bun_G, along with its twisted versions $\mathfrak{D}\text{-mod}_\kappa(\mathrm{Bun}_G)$, where κ is a *level* (what this means will be explained in the subsequent lectures).

The notion of D-module on an algebraic stack will be discussed in Sam Raskin's talk next week. Properly speaking, instead of considering the abelian categories $\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)$ (resp., $\mathfrak{D}\text{-mod}_\kappa(\mathrm{Bun}_G)$), we should be considering the corresponding derived categories, denoted $D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G))$ (resp., $D(\mathfrak{D}\text{-mod}_\kappa(\mathrm{Bun}_G))$). These can be defined using the appropriate homotopy category apparatus, but this discussion will be postponed until the next semester. For now, whenever these derived categories make an appearance, you should assume their existence as triangulated categories with the reasonable functorial properties.

The "classical" (as opposed to "quantum") Geometric Langlands aims to study the behavior of the category $\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)$ under a certain large commutative family of functors that act on it, called the Hecke functors.

1.1. **The case $G = \mathbb{G}_m$.** We shall first consider the case of the simplest reductive group G, namely the multiplicative group $\mathbb{G}_m$. In this case we identify Bun_G with $\mathrm{Pic}(X)$–the Picard stack of X, i.e., the stack classifying line bundles on X.

For a point x we have the natural map

$$m_x : \mathrm{Pic}(X) \to \mathrm{Pic}(X),$$

that sends a line bundle $\mathcal{L}$ to the line bundle $\mathcal{L}(x)$, i.e., we add to a line bundle the divisor corresponding to the point x. We consider the functor $m_x^* : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G))$.

We can also allow the point x move along X, and we obtain a map

$$m : X \times \mathrm{Pic}(X) \to \mathrm{Pic}(X),$$

and the corresponding functor $m^* : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)) \to D(\mathfrak{D}\text{-mod}(X \times \mathrm{Bun}_G))$.

The functors m_x^*, $x \in X$ and m^* are the simplest examples of Hecke functors (respectively, local and global).

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1.2. **The case of $G = GL_n$.** For $G = GL_n$ we can think of Bun_G as the stack Bun_n classifying rank n vector bundles on X. However, given a point $x \in X$ there is no distinguished way to map Bun_n to itself, as was the case with the map m_x. Instead, we have a correspondence:

$$\mathrm{Bun}_n \xleftarrow{\overleftarrow{h}} \mathcal{H}_x \xrightarrow{\overrightarrow{h}} \mathrm{Bun}_n,$$

where $\mathcal{H}_x$ is the stack classifying triples

$$(\mathcal{M}, \mathcal{M}', \alpha : \mathcal{M} \hookrightarrow \mathcal{M}' \,|\, \mathcal{M}/\mathcal{M}' \text{ is of length 1 and supported at } x \in X).$$

The maps $\overleftarrow{h}$ and $\overrightarrow{h}$ send a triple $(\mathcal{M}, \mathcal{M}', \alpha) \in \mathcal{H}_x$ as above to $\mathcal{M} \in \mathrm{Bun}_n$ and $\mathcal{M}' \in \mathrm{Bun}_n$, respectively. Note that for $n = 1$, both maps $\overleftarrow{h}$ and $\overrightarrow{h}$ are isomorphisms and the resulting map $\mathrm{Bun}_1 \to \mathrm{Bun}_1$ is exactly m_x.

We define the functor $H_x : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n))$ by

$$\mathcal{F} \mapsto \overleftarrow{h}_! \circ \overrightarrow{h}^*(\mathcal{F})[n-1]$$

(the necessity for the cohomological shift $[n-1]$ will become clear later).

1.3. **Other Hecke functors.** Let us note, however, that the stack $\mathcal{H}_x$, and the functor H_x that came along with it, accounted for the "minimal" way to modify a vector bundle at a point x, but not for the only way. Here are some other possibilities.

For $i = 1, ..., n$, define $\mathcal{H}_x^{\Lambda^i}$ to be the stack classifying triples

$$(\mathcal{M}, \mathcal{M}', \alpha : \mathcal{M} \hookrightarrow \mathcal{M}' \,|\, \mathcal{M}/\mathcal{M}' \text{ is of length } i \text{ and supported } \textit{scheme-theoretically} \text{ at } x \in X)$$

(where for $i = 1$ we recover $\mathcal{H}_x$). Denote the corresponding functor

$$\mathcal{F} \mapsto \overleftarrow{h}_! \circ \overrightarrow{h}^*(\mathcal{F})[(n-i) \cdot i] : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n))$$

by $H_x^{\Lambda^i}$.

For $m = 1, 2, ...$ define $\mathcal{H}_x^{\mathrm{Sym}^m}$ to be the stack classifying triples

$$(\mathcal{M}, \mathcal{M}', \alpha : \mathcal{M} \hookrightarrow \mathcal{M}' \,|\, \mathcal{M}/\mathcal{M}' \text{ is of length } m \text{ and supported } \textit{set-theoretically} \text{ at } x \in X)$$

(where for $k = 1$ we again recover $\mathcal{H}_x$). Denote the corresponding functor

$$\mathcal{F} \mapsto \overleftarrow{h}_! \circ \overrightarrow{h}^*(\mathcal{F})[(n-1) \cdot m] : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n))$$

by $H_x^{\mathrm{Sym}^m}$.

Define the functor $H_x^{\mathrm{Sym}^m \oplus \Lambda^i} : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n))$ as $H_x^{\mathrm{Sym}^m} \oplus H_x^{\Lambda^i}$. Define the functor $H_x^{\mathrm{Sym}^m \otimes \Lambda^i} : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n))$ as $H_x^{\mathrm{Sym}^m} \circ H_x^{\Lambda^i}$, and so on.

From the above, we see that it is natural to consider *the category of Hecke functors* attached to the point $x \in X$, which act from $D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_n))$ to itself, and which

$$H_x, H_x^{\Lambda^i}, H_x^{\mathrm{Sym}^m}, H_x^{\mathrm{Sym}^m \oplus \Lambda^i}, H_x^{\mathrm{Sym}^m \otimes \Lambda^i}, \text{ etc.}$$

are examples of. Moreover, this category is supposed to have a natural structure of monoidal category.

For an arbitrary group G, this category will be defined in the talks about the affine Grassmannian and the Geometric Satake Equivalence. This category will be denoted $Hecke_{G,x}$. Moreover, the following theorem will be proved:

Theorem 1.4. [Drinfeld, Ginzburg, Lusztig, Mirkovic-Vilonen]
The monoidal category $Hecke_{G,x}$ is naturally a tensor category, and as such it is canonically equivalent to the category of finite-dimensional representations of a reductive group.

The reductive group that appears in the formulation of the above theorem is called "the Langlands dual group pf G", and denoted $\check{G}$. The remarkable fact is that it can be described completely combinatorially in terms of G: its root system is the dual one to that of G.

Thus, to $x \in X$ and $\mathbf{a} \in \mathrm{Rep}(\check{G})$ we can associate a functor

$$H_x^{\mathbf{a}} : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)) \to D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)),$$

and composition of such functors corresponds to the tensor product of the V's.

Moreover, as in the case of H_x for GL_n, we can let the point x move, and we obtain the global Hecke functor

$$H^{\mathbf{a}} : D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G)) \to D(\mathfrak{D}\text{-mod}(X \times \mathrm{Bun}_G)).$$

1.5. **Eigenvectors and eigenvalues.** Let A be a commutative algebra acting on a vector space W. In this case we can talk about eigenvectors and eigenvalues. An eigenvalue is a point of $\mathrm{Spec}(A)$, i.e., a homomorphism $\sigma : A \to k$. An eigenvector with eigenvalue σ is an element $w \in W$ satisfying $a(w) = \sigma(a) \cdot w$, $\forall a \in A$.

Let now $\mathbf{A}$ be a tensor category acting on a category $\mathbf{W}$; we assume that $\mathbf{W}$ is k-linear, i.e., it makes sense to tensor objects of $\mathbf{W}$ by k-vector spaces, which are the categorical replacement of scalars. By an eigenvalue we shall mean a tensor functor $\sigma : \mathbf{A} \to \mathrm{Vect}_k$. An eigenobject with eigenvalue σ is by definition an object $\mathbf{w} \in \mathbf{W}$ *endowed with a family of of isomorphisms*

$$\gamma_{\mathbf{a}} : \mathbf{a}(\mathbf{w}) \simeq \sigma(\mathbf{a}) \otimes \mathbf{w},\ \forall \mathbf{a} \in \mathbf{A}.$$

Note, however, that there is one more thing we could (and should) ask for. Namely, let $\mathbf{a}_1$ and $\mathbf{a}_2$ be two objects of $\mathbf{A}$. Then we have two isomorphisms

$$\mathbf{a}_1(\mathbf{a}_2(\mathbf{w})) \rightrightarrows \sigma(\mathbf{a}_1) \otimes \sigma(\mathbf{a}_2) \otimes \mathbf{w},$$

given by the two circuits of the diagram:

$$\begin{tikzcd}
\mathbf{a}_1(\mathbf{a}_2(\mathbf{w})) \arrow[r, "\sim"] \arrow[d, "\mathbf{a}_1(\gamma_{\mathbf{a}_2})"'] & (\mathbf{a}_1 \otimes \mathbf{a}_2)(\mathbf{w}) \arrow[d, "\sigma_{\mathbf{a}_1 \otimes \mathbf{a}_2}"'] \\
\mathbf{a}_1(\sigma(\mathbf{a}_2) \otimes \mathbf{w}) \arrow[d, "\sim"'] & (\mathbf{a}_1 \otimes \mathbf{a}_2) \otimes \mathbf{w} \arrow[d, "\sim"'] \\
\sigma(\mathbf{a}_2) \otimes (\mathbf{a}_1(\mathbf{w})) \arrow[r, "\gamma_{\mathbf{a}_1}"] & \sigma(\mathbf{a}_2) \otimes \sigma(\mathbf{a}_1) \otimes \mathbf{w},
\end{tikzcd}$$

and we ask that this diagram be commutative.

1.6. **Hecke eigensheaves.** We apply the above discussion to

$$\mathbf{A} = \mathrm{Rep}(\check{G}) \text{ and } \mathbf{W} = D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G))$$

where the action is given by the functors $\mathbf{a} \mapsto H_x^{\mathbf{a}}$ for a fixed point $x \in X$.

First, what are the possible eigenvalues? By the above discussion, eigenvalues are given by tensor functors $\mathrm{Rep}(\check{G}) \to \mathrm{Vect}_k$, which are the same as $\check{G}$-torsors over a point.

However, rather than considering the functors $H^{\mathbf{a}}_x$ for all $x \in X$ separately, we should rather consider the global Hecke functors $H^{\mathbf{a}}$. In this setting, eigenvalues should be *families of $\check{G}$-torsors, parametrized by points of* X, and such an object is otherwise known as a $\check{G}$-local system on X, or, equivalently, a tensor functor

$$\mathrm{Rep}(\check{G}) \to \mathrm{LocSys}(X),$$

where $\mathrm{LocSys}(X)$ is the category of *local systems=$\mathcal{O}$-coherent D-modules=vector bundles with a (flat) connection* on X.

Definition 1.7. *For a fixed local system σ, a Hecke eigensheaf with respect to σ is an object $\mathcal{F}_\sigma \in D(\mathfrak{D}\text{-mod}(\mathrm{Bun}_G))$ endowed with a family of isomorphisms*

$$\gamma_{\mathbf{a}} : H^{\mathbf{a}}(\mathcal{F}_\sigma) \simeq \sigma(\mathbf{a}) \boxtimes \mathcal{F}_\sigma \in D(\mathfrak{D}\text{-mod}(X \times \mathrm{Bun}_G)),$$

where $\sigma(\mathbf{a})$ is the corresponding ($\mathcal{O}$-coherent) D-module.

Warning: the above definition is a provisional one, as we have omitted to impose two requirements on the isomorphisms γ. One requirement is the compatibility with the monoidal structure, given by the commutativity of a diagram similar to the one above. Another requirement is something we have not yet touched upon. Namely, it has do to with the interaction of the functors $H^{\mathbf{a}_1}_{x_1}$ and $H^{\mathbf{a}_2}_{x_2}$ for distinct points $x_1, x_2 \in X$; we shall get back to this discussion in future talks.

One of the goals of Geometric Langlands was to find solutions to the eigensheaf problem. Namely, given a $\check{G}$-local system σ, can we construct a (non-zero) Hecke eigensheaf $\mathcal{F}_\sigma$ with respect to it?

The [BD] book we are studying gives a construction of such $\mathcal{F}_\sigma$ for a specific class of $\check{G}$-local systems called "opers".

1.8. **Back to $G = \mathbb{G}_m$.** For $G = \mathbb{G}_m$ it will turn out that $\check{G} = G$ and a $\check{G}$-local system on X is the same as a 1-dimensional local system E (=$\mathcal{O}$-coherent D-module of rank 1) on X. The full definition of a Hecke eigensheaf will be equivalent to the following one:

Definition 1.9. *A Hecke eigensheaf with respect to E is an object $\mathcal{F}_E \in D(\mathfrak{D}\text{-mod}(\mathrm{Pic}(X)))$ endowed with an isomorphism $\gamma : m^*(\mathcal{F}_E) \simeq E \boxtimes \mathcal{F}_E$, such that the following condition holds:*

Consider the map $m_2 : X \times X \times \mathrm{Pic}(X) \to \mathrm{Pic}(X)$. The isomorphism γ gives rise to an isomorphism

$$\gamma_2 : m_2^*(\mathcal{F}_E) \simeq E \boxtimes E \boxtimes \mathcal{F}_E.$$

Moreover, both sides in the above isomorphisms are naturally Σ_2-equivariant, with respect to the natural action of Σ_2 that swaps the two copies of X.

Our condition is that the map γ_2 be also Σ_2-equivariant, i.e., compatible with the equivariant structures on the two sides.

What about the existence of $\mathcal{F}_E$ for a given E? This is easy in this case. Namely, let $\mathrm{Pic}^1(X)$ be the connected component of $\mathrm{Pic}(X)$ corresponding to line bundles of degree 1. Using the fact that the canonical map $X \to \mathrm{Pic}^1(X)$ induces an isomorphism

$$(\pi_1(X))^{ab} \simeq \pi_1(\mathrm{Pic}^1(X)),$$

given E, we can construct a 1-*dimensional local system* $\mathcal{F}_E$ on $\mathrm{Pic}^1(X)$, and spread it to other connected components of $\mathrm{Pic}(X)$ using the Hecke eigen-property.

Unfortunately, nothing as simple would work for a non-abelian reductive group G. In particular, $\mathcal{F}_\sigma$ will not be a local system (=$\mathcal{O}$-coherent). However, in some favorable cases (such as one discussed in the book), it will be a *D-module*, rather than an object of the derived category.

2. Hitchin's system and its quantization

2.1. **The case $G = \mathbb{G}_m$.** It will turn out that opers for $\mathbb{G}_1$ are those 1-dimensional local systems E, whose underlying line bundle (recall that E amounts to a line bundle with a connection) is trivial. I.e., specifying E is equivalent to specifying a 1-form ν on X.

In this case, we can interpret the construction of $\mathcal{F}_E$ as follows. Recall that the map $X \to \mathrm{Pic}(X)$ also induces an isomorphism

$$H^0(\mathrm{Pic}(X), \Omega^1_{\mathrm{Pic}(X)}) \to H^0(X, \Omega_X),$$

and in addition we have

$$H^0(\mathrm{Pic}(X), \Omega^1_{\mathrm{Pic}(X)}) \simeq T^*_e(\mathrm{Pic}(X)) \simeq \text{ translation-invariant 1-forms on } \mathrm{Pic}(X).$$

So, starting with a 1-form ν on X, we can produce an invariant (and, hence, closed) 1-form on $\mathrm{Pic}(X)$, and thus a flat connection on the trivial line bundle. This is our $\mathcal{F}_E$.

2.2. **D-modules on a commutative group.** Let us conceptualize the above construction slightly differently. Let H be a commutative group (or commutative group-stack). We have

$$T^*H \simeq H \times \mathfrak{h}^*,$$

so we have a canonical map $T^*H \to \mathfrak{h}^*$, which can be thought of as a map of algebras

$$\mathrm{Sym}(\mathfrak{h}) \to \Gamma(T^*H, \mathcal{O}_{T^*H}).$$

The image of $\mathrm{Sym}(\mathfrak{h})$ is a subalgebra consisting of elements that Poisson-commute with each other (for the Poisson structure on T^*H coming from the canonical symplectic structure).

Definition 2.3. *A classical completely integrable system on a smooth symplectic scheme Y is a Poisson-commuting subalgebra $A^{cl} \subset \Gamma(Y, \mathcal{O}_Y)$, such that $2\dim(A^{cl}) = \dim(Y)$.*

The above picture has a "quantum" analog, which is what is of primary interest for us, when $Y = T^*Z$ for a smooth scheme Z. Namely, we regard the algebra of differential operators on Z as a quantization of $\mathcal{O}_{T^*Z}$.

In our case $Y = T^*Z$, where Z is actually a stack, so one has to be careful with (a) smoothness, (b) being symplectic, (c) dimension and (d) differential operators. But it will work out at the end.

Definition 2.4. *A quantum completely integrable system on a smooth scheme Z is a commutative subalgebra $A \subset \Gamma(Z, \mathfrak{D}_Z)$, such that $\dim(A) = \dim(Z)$.*

Definition 2.5. *We say that a QCIS $A \subset \Gamma(Z, \mathfrak{D}_Z)$ is the quantization of $A^{cl} \subset \Gamma(Y, \mathcal{O}_Y)$ (or, equivalently, the latter is the quasi-classics of the former) if $A^{cl} = \mathrm{gr}(A)$, with respect to the filtration induced by the canonical filtration on $\mathfrak{D}_Z$.*

Note that in the case of Z being a commutative group H, the above CCIS admits a natural quantization, given by the map

$$\mathrm{Sym}(\mathfrak{h}) \simeq U(\mathfrak{h}) \to \Gamma(H, \mathfrak{D}_H),$$

where $\mathfrak{h}$ maps maps identically to translation-invariant vector fields on H.

In general, whenever we have a QCIS system $A \subset \Gamma(Z, \mathfrak{D}_Z)$, and a point $\sigma \in \operatorname{Spec}(A)$, we can attach to it a D-module $\mathcal{F}_\sigma$ on Z by

$$\mathcal{F}_\sigma := \mathfrak{D}_Z \underset{A}{\otimes} k,$$

where $A \to k$ is the homomorphism corresponding to σ.

The above construction $E \mapsto \mathcal{F}_E \in \mathfrak{D}\text{-mod}(\operatorname{Pic}(X))$ is a particular case of the above paradigm.

2.6. **The [BD] ansatz.** For general G, of course Bun_G does not have a group structure. However, we will still have a classical integrable system

$$A^{cl} \to \Gamma(T^* \operatorname{Bun}_G, \mathcal{O}_{T^* \operatorname{Bun}_G}),$$

and its quantization

$$A \to \Gamma(\operatorname{Bun}_G, \mathfrak{D}'_{\operatorname{Bun}_G}),$$

with a slight difference that instead of ordinary differential operators $\mathfrak{D}_{\operatorname{Bun}_G}$ we shall be dealing with a ring of twisted differential operators, denoted $\mathfrak{D}'_{\operatorname{Bun}_G}$.

The corresponding classical integrable system is called the Hitchin system and is relatively easy to describe (see below). The quantization is much more tricky. The situation can be summarized as follows:

- (i) There exists a commutative subalgebra $A \subset \Gamma(\operatorname{Bun}_G, \mathfrak{D}'_{\operatorname{Bun}_G})$.
- (ii) The scheme $\operatorname{Spec}(A)$ is naturally a closed subscheme inside the stack $\operatorname{LocSys}_{\check{G}}$ that classifies $\check{G}$-local systems on X.
- (iii) For $\sigma \in \operatorname{Spec}(A)$ the corresponding twisted D-module $\mathcal{F}_\sigma := \mathfrak{D}'_{\operatorname{Bun}_G} \underset{A}{\otimes} \mathbb{C}$ has a structure of Hecke eigensheaf with respect to σ, when the latter is viewed as a local system via (ii).

2.7. **The classical Hitchin system.** The affine scheme $\operatorname{Spec}(A^{cl})$ is called the Hitchin base and denoted $\operatorname{Hitch}(X)$. The Hitchin integrable system is therefore a map $\mu : T^* \operatorname{Bun}_G \to \operatorname{Hitch}(X)$.

For simplicity, we shall consider the case $G = GL_n$; the general case its similar. For GL_n

$$\operatorname{Hitch}(X) \simeq H^0(X, \Omega_X) \times H^0(X, \Omega_X^{\otimes 2}) \times ... \times H^0(X, \Omega_X^{\otimes n}).$$

Recall also that $T^* \operatorname{Bun}_n$ can be thought as the moduli space of pairs $(\mathcal{M}, f)$, where $\mathcal{M}$ is a rank n vector bundle on X, and f is a map $\mathcal{M} \to \mathcal{M} \otimes \Omega_X$. The required map assigns to a pair $(\mathcal{M}, f)$ as above and $i = 1, ..., n$ the *trace* of f^i, which is an element of $\Gamma(X, \Omega_X^{\otimes i})$, where f^i is the composition

$$\mathcal{M} \xrightarrow{f} \mathcal{M} \otimes \Omega_X \xrightarrow{f \otimes \mathrm{id}} \mathcal{M} \otimes \Omega_X^{\otimes 2} \to ... \to \mathcal{M} \otimes \Omega_X^{\otimes n}.$$

2.8. **The local to global principle.** Although the classical Hitchin system is easy to define, its quantization is not, and the verification of the eigensheaf property is even less so. The construction of the quantization is based on a local-to-global principal, which involves choosing a point $x \in X$, and then letting it move, and some infinite-dimensional representation theory.

We shall consider a bigger stack, namely, $\operatorname{Bun}_G^{level_x}$, which classifies G-bundles with a full level structure at x. This stack is acted on by the (infinite-dimensional) groups $G(\mathcal{O}_x) \subset G(\mathcal{K}_x)$, where $\mathcal{O}_x$ (resp., $\mathcal{K}_x$) is the completed local ring (resp., field) of X at x. We have

$$\operatorname{Bun}_G \simeq \operatorname{Bun}_G^{level_x} / G(\mathcal{O}_x).$$

From this picture we shall be able to reinterpret the Hitchin map μ as follows. We shall introduce the local Hitchin space Hitch_x. E.g., for $G = GL_n$,

$$\mathrm{Hitch}_x := \Omega_{\mathcal{O}_x} \times \Omega_{\mathcal{O}_x}^{\otimes 2} \times ... \times \Omega_{\mathcal{O}_x}^{\otimes n},$$

and the moment map for the action of $G(\mathcal{K}_x)$ will give rise to the local Hitchin map

$$\mu_x : T^* \mathrm{Bun}_G \to \mathrm{Hitch}_x,$$

such that $\mu_x = \iota_x \circ \mu$, where ι_x is the map

$$\mathrm{Hitch}_x \hookrightarrow \mathrm{Hitch}(X),$$

given by the Taylor expansion.

The picture with Hitch_x and μ_x will admit a quantization, eventually giving rise to the algebra A.