MATH 224 (SPRING, 2018), PROGRAM

Week 1.

Day 1 (Tue, Jan 23):

1) Definition of algebraic groups:
(a) As a functor of points;
(b) Hopf algebras;
(c) Examples.

2) Actions and representations:
(a) Action of groups on schemes;
(b) Algebraic representations;
(c) Every rep is a union of f.d.

Day 2 (Thurs, Jan 25):

1) Every rep is a union of fin. dim.:
(a) Proof;
(b) Embedding into GL_n;
(c) Embedding of affine G-varieties into linear ones;

2) The regular representation:
(a) Adjunction;
(b) Presentation as a colimit (beginning).

Week 2.

Day 1 (Tue, Jan 30):

No class–was sick.

Date: February 6, 2018.

Day 2 (Thurs, Feb. 1):

1) Examples:
(a) Reps of $\mathbb{G}_a$;
(b) Reps of finite groups;
(c) Reps of $\mathbb{G}_m$;
(d) Tori.

2) Tannaka's theorem:
(a) Monoidal functors;
(b) Statement: description of S-points;
(c) Proof via the regular representations;
(d) Tensoring with the regular representation.

Week 3.

Day 1 (Tue, Feb 6):

1) The neutral connected component:
(a) Stated and proved theorem about the neutral connected component;
(b) Examples of connected and disconnected groups;
(c) Generating a (connected) subgroup by images of maps of irreducible varieties.

2) "Generic $\Rightarrow$ everywhere" argument:
(a) A homomorphism of algebraic groups is onto its image;
(b) An orbit of an action of an algebraic group its open in its closure;
(c) Side remark: a map of algebraic groups with a finite kernel is finite.

3) The notion of transitivity of action:
(a) A naively transitive action defines a fully faithful map;
(b) Failure of smoothness;
(c) The Frobenius map (beginning).