Lecture 4: 16/9/20

$LG = \mathrm{colim}_a X_a$, $n \in \mathbb{N}$ //

Thm'': $\forall a, n \ \exists N(a,n)$ s.t. $\forall w \in \tilde{W}$, $\forall \alpha \in J_w \subseteq \tilde{\Delta}$

$w(\alpha) > 0 \Uparrow$

$w(\alpha) - N(a,n) \geq 0$, $\forall J \subseteq J_w - \{\alpha\} \subsetneq \tilde{\Delta}$

$$\boxed{\mathrm{ind}_{W_J}^{W_{J\cup\alpha}} \left(S^{wW_J}_{W_J} \right) \longrightarrow S^{wW_{J\cup\alpha}}_{W_{J\cup\alpha}}}$$ is $\pi_{a,n}$-equiv,

i.e. become iso after $(- \otimes \omega_{X_a}) * 1_{I_n}$.

$S^Y = p^Y_*(\Omega_Y)$ $\qquad p^Y : LG^Y \times^I I \longrightarrow LG$

$(g, x) \longmapsto g x g^{-1}$

$Y \subseteq \tilde{W}$ finite ; $S = \mathrm{colim}_Y S^Y$ $\qquad$ | $J \subsetneq \tilde{\Delta}$

$$\begin{tikzcd} 0 \arrow[r] & 0 \\ X \arrow[u] \arrow[r, "\sim"] & Y \arrow[u, "\neg"'] \end{tikzcd}$$

particular case: $J = \emptyset$

$\forall a, n, \ \exists N(a,n) \ \forall w \ \forall \alpha$

s.t. $w(\alpha) > N(a,n) > 0$,

$$\boxed{\mathrm{ind}_1^{W_\alpha}(S^w) \longrightarrow S^{wW_\alpha}_{W_\alpha}}$$ is $\pi_{a,n}$-equiv.

Claim: part case follows from

Lemma: (a) If $w(\alpha) \gg_{X_a} 0$, then $\qquad X_a = IwI$

$X_a \cap w P_\alpha w^{-1} = X_a \cap w I w^{-1}$, $\quad P_\alpha \supseteq I$ min par corr to α.

(b) $(wI^+w^{-1}) \cdot I_n = (w P_\alpha^+ w^{-1}) I_n$, if $w(\alpha) > n$

$I \to T \xrightarrow{\alpha} \mathbb{G}_m$; $I = \underset{(I - I^+)}{\{\alpha \neq 1\}} \cup \underset{I^+}{\{\alpha = 1\}}$

$\widetilde{LG}^\gamma = LG^\gamma \times^I I$

$\widetilde{LG}^\gamma_{tu} := LG^\gamma \times^I I^+$

$\widetilde{LG}^\gamma_{rss} = LG^\gamma \times^I (I - I^+)$

$\Rightarrow \quad S^\gamma_{tu} \to S^\gamma \to S^\gamma_{rss} \to$

Enough to show

$$\left.\begin{array}{l} (1)\ \mathrm{ind}_1^{W_\alpha}(S^W_{rss}) \to S^{WW_\alpha}_{W_\alpha, rss} \\ (2)\ \mathrm{ind}_1^{W_\alpha}(S^W_{tu}) \to S^{WW_\alpha}_{W_\alpha, tu} \end{array}\right\} \underline{\Pi_{a,n}\text{-equiv.}}$$

actually,

$$(1')\ \mathrm{ind}_1^{W_\alpha}(S^W_{rss}) \overset{!\Omega}{\otimes} X_a \xrightarrow{\sim} S^{WW_\alpha}_{W_\alpha, rss} \overset{!}{\otimes} d_{X_a}$$

$w(\alpha) \gg 0$
X_a.

pf: enough to show iso of !-stalks at $\gamma \in X_a$, i.e.

$$\mathrm{ind}_1^{W_\alpha} H_*(Fl^W_\gamma) \underset{W_\alpha\text{-repr}}{\xrightarrow{\sim}} H_*(Fl^{WW_\alpha}_\gamma) \quad \forall \gamma \in LG$$

$\underline{+ \text{reg s.s.}}$

$\Uparrow$

$$Fl^{WW_\alpha}_\gamma = Fl^W_\gamma \underset{s_\alpha}{\sqcup} Fl^{Ws_\alpha}_\gamma \Leftarrow$$

Remark: $w(\alpha) > 0 \iff Fl^w \xrightarrow{\sim} Fl^w_{P_\alpha}$

$ \overset{\shortparallel}{Iw} \qquad \overset{\shortparallel}{Iw}$

Lemma: $w(\alpha) \gg_\gamma 0 \quad Fl^w_\gamma \xrightarrow{\sim} Fl^w_{P_\alpha, \gamma}$

$\{g \mid g^{-1}\gamma g \in I\} \quad \{g \mid g^{-1}\gamma g \in P_\alpha\}$

γ-rss $\quad Fl^{ws_\alpha}_\gamma \longrightarrow Fl^{ws_\alpha}_{\gamma, P_\alpha}$ is s_α-torsor, if γ has rss reduction

$\phantom{\gamma\text{-rss}} \cup \qquad \nearrow \sim$

$\phantom{\gamma\text{-rss}} Fl^w_\gamma$

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(2) $\to$ (2a) $\quad S^{w}_{tw} \overset{!}{\otimes} \omega_{X_\alpha} \xrightarrow{\sim} (S^{ws_\alpha}_{tw})_{s_\alpha} \overset{!}{\otimes} X_\alpha$

$w(\alpha) \gg 0.$

(2) $\to$ (2b) $\quad S^w_{tw} * 1_{I^u} \xrightarrow{\sim} (S^{ws_\alpha}_{tw})_{s_\alpha, sgn} * 1_{I^u}$

$w(\alpha) \geq n$

(2a) $\forall \gamma \in X_\alpha$ calculate !-stacks

$$H_*(Fl^w_\gamma) \underset{\pi}{\xrightarrow{\sim}} H_*(Fl^w_{\gamma, P_\alpha})$$

$(Spr_{SL_2})_W \simeq \mathbb{Q}_e$

$(Spr_{SL_2})_{W, sgn} = \delta_e$

$Av^I(\delta_{wI^+w^{-1}}) * \delta_{I_n}$

$\shortparallel$

$Av^I(\delta_{I P_\alpha w^{-1}}) * \delta_{I_n}$

$J = \phi$ graded pieces are

$w=1$ $S^w \xrightarrow{\sim} S^w$ Id

$w=s_2$ $S^w \to S^{ws_2}$!

Cor of pf: $(S)_{\Xi=1} \xrightarrow{\sim} \mathcal{M}_{LG_c}$; $S = (p_! p^! \mathcal{M}_{LG_c})_{\Xi=1}$

Pf: Want it is isomorphism $\mathcal{M}_{LG_c}$

enough $\forall \gamma \in LG$, $\Rightarrow$ May assume $\gamma \in I$

$S_{\tilde{w}} = \operatorname{colim}_J S_{w_J} = \operatorname{colim}_J \operatorname{colim}_{\lambda} S_{w_J}^{\gamma w_J}$

$\qquad\qquad\qquad\qquad\qquad\qquad \operatorname{colim}_{\gamma} \left(\operatorname{colim}_J S_{w_J}^{\gamma w_J}\right)$

Claim: $\left(\operatorname{colim}_J S_{w_J}^{\gamma w_J}\right) \otimes \mathcal{M}_I \xrightarrow{\sim} \mathcal{M}_I$. $\forall \gamma$.

Cor 2 of pf: $(S_{sgn})|_{LG_{tu}} * 1_{I_n}$ - compact $\forall n$.

$(S_{sgn})|_{LG_{tu}} * 1_{I^+} = \delta_{I^+}$