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G - reductive group

$\check{G}$ - Langlands dual

Λ - coweight lattice of G

$$q : \Lambda \longrightarrow \mathbb{C}^*$$

W-invariant

ℓ-adic sheaves $\mathbb{C}^* \longrightarrow \overline{\mathbb{Q}}_\ell(-1)$

D-modules $\mathbb{C}^* \longrightarrow k/\mathbb{Z}$

In general, the value

group is Kummer sheaves on $\mathbb{G}_m$.

$q \longrightarrow$ gerbe on Gr_G, Fl_G

(Lysenko - G)

$$Shv_q(Gr_G)^{(N(K),\chi)} =: Whit_q(Gr_G)$$

$$Shv_q(Fl_G)^{(N(K),\chi)} =: Whit_q(Fl_G)$$

Gerbe on Gr_G, Fl_G is $N(K)$-equiv

Have a gerbe on $G(K)$, splits canonically over $N(K)$

$Rep_q(\check{G})$ - f.d. representations of Lusztig's quantum group

$\mathrm{Rep}_q(\check{G})^{\text{mixed}}$

$U_q(N^+)^{\text{Lus}}$

$U_q(N^-)^{DK}$ – Hopf dual.

$\Uparrow$

Hopf algebras $\quad \mathrm{Rep}_q(\check{T})$

$\qquad\qquad\qquad\qquad \simeq$

$\qquad\qquad\qquad \mathrm{Vect}^{\Lambda}$

q defines a braiding

$\to \; U_q(N^+)^{\text{Lus}}$

$\quad\; U_q(N^-)^{DK} \curvearrowright M$

$\qquad\qquad\qquad\qquad \Uparrow$

$\qquad\qquad\qquad \mathrm{Rep}_q(\check{T})$

$$Z_{\mathrm{Rep}_q(\check{T}),\, D_r}\left(U_q(N^+)\text{-mod}_{\text{loc.nilp}}\right)$$

$$\|\,?$$

$$\mathrm{Rep}_q(\check{G})^{\text{mixed}}$$

$$Z_{\mathrm{Rep}(\check{T})}\left(\mathrm{Rep}(N^+)\right)$$

$$\|$$

$$\mathrm{QCoh}(\check{N}/\mathrm{Ad}(\check{B}))$$

$$\begin{tikzcd}
\mathrm{Whit}_q(\mathrm{Gr}_G) \arrow[rr, leftrightarrow, bend left, "FLE"] \arrow[rr, phantom, "{[CDR]}"'] \arrow[dr, leftrightarrow, "\text{Lurie's original case}"'] & & \hat{\mathfrak{g}}\text{-mod}_{\check{\kappa}}^{\check{G}(O)} \arrow[dl, leftrightarrow, "KL"] \\
& \mathrm{Rep}_q(\check{G}) &
\end{tikzcd}$$

$$
\begin{tikzcd}
\mathrm{Whit}_{q}(\mathrm{Fl}_G) \arrow[rr, leftrightarrow, bend left, "\mathrm{FLE}", "{[\mathrm{CDR}]}"'] \arrow[dr, "\simeq"] \arrow[ddr, bend right, "\mathrm{R.\,Yang}\ \wr\,?"'] & & \hat{\mathfrak{g}}\text{-mod}_{\kappa'}^{\check{I}} \arrow[dl, "\sim"'] \arrow[ddl, leftrightarrow, bend left, "\mathrm{Chen\text{-}Fu}"] \\
& \Omega_q^{\mathrm{Lus}}\text{-mod}^{\mathrm{Fact}} \arrow[d, leftrightarrow] & \\
& \mathrm{Rep}_q(\check{G})^{\mathrm{mixed}} & {[\mathrm{BFS}]} \\
& & \mathrm{Lurie} \arrow[u]
\end{tikzcd}
$$

$\searrow$: Ruotao Yang

$\swarrow$ dR $\qquad \updownarrow$ Betti

$$\mathrm{Conf} = \bigsqcup_{\lambda \in \Lambda^{neg}} \mathrm{Conf}^{\lambda}$$

. .

$\sum \lambda_i x_i \qquad \lambda_i \in \Lambda^{wy}$

$\qquad\qquad\qquad x_i \in X$

$x \in X$

$\mathrm{Conf}_{\infty \cdot x}$ – we allow $\lambda \cdot x$, $\lambda \in \Lambda$

$$\begin{tikzcd}
(\mathrm{Conf} \times \mathrm{Conf})_{disj} \subseteq \mathrm{Conf} \times \mathrm{Conf} \arrow[d, "add"] \\
\mathrm{Conf}
\end{tikzcd}$$

$q \longrightarrow$ gerbe on Conf

$\Omega \in \mathrm{Shv}_q(\mathrm{Conf})$

$$\mathrm{add}^!(\Omega)\big|_{\mathrm{disj}} \simeq \Omega \boxtimes \Omega\big|_{\mathrm{disj}}$$

$\Omega\text{-mod}^{\mathrm{Fact}}$

$\mathcal{F} \in \mathrm{Shv}_q(\mathrm{Quot}_{\infty \cdot x})$

$$\mathrm{add}^!(\mathcal{F})\big|_{\mathrm{disj}} \simeq \Omega \boxtimes \mathcal{F}\big|_{\mathrm{disj}}$$

$\mathrm{Quot} \times \mathrm{Quot}_{\infty x}$

Lurie / Beilinson realized

that factorization algebras

$\Uparrow$

E_n-algebras

Thm (a) Hopf algebras in Λ^{pos}

$$\Updownarrow \quad \mathrm{Rep}_q(\check{T})$$

factorization algebras in $\mathrm{Shv}_{\mathcal{G}}(\mathrm{Gr})$

shift-invariant. (der $\mathbb{A}^1$)

(b) $\underset{\mathrm{Rep}_q(\check{T}),\,\mathrm{Dr}}{Z}\left(H\text{-mod}_{\mathrm{loc.nilp}}\right) = \Omega_H\text{-mod}^{\mathrm{Fact.}}$

$$H \longleftrightarrow \Omega_H$$

$$\lambda \cdot x \xrightarrow{i_{\lambda \cdot x}} \mathrm{Gr}$$

$$i^!_{\lambda x}(\Omega) = (C^\cdot(H))^\lambda$$

$$i^*_{\lambda x}(\Omega) = (C_\cdot(H^\vee))^\lambda$$

$$M \longleftrightarrow \mathcal{F}$$

$$i^!_{\lambda \cdot x}(\mathcal{F}) = C^\cdot(H, M)^\lambda$$

$$\check{C}^*_{\lambda,x}(\mathcal{F}) = C_\cdot(H^\vee, M)^\lambda$$

$$H = U_q(\check{N}^+)^{\mathrm{lus}}$$

$$\Omega_q^{\mathrm{lus}}$$

$$\mathrm{Rep}_q(\check{G})^{\mathrm{mixed}} \simeq \Omega_q^{\mathrm{lus}}\text{-mod}^{\mathrm{Fact.}}$$

$$\begin{tikzcd}
\mathrm{Rep}_q(\check{G})^{\mathrm{mixed}} \arrow[r, bend left, "J_!"] \arrow[r, "J_{!*}"] \arrow[r, bend right, "J_*"'] & \mathrm{Rep}_q(\check{T})
\end{tikzcd}$$

$$M \longrightarrow C^\cdot(U_q(N^+)^{\mathrm{lus}}, M)$$

$$M \longrightarrow C^\cdot(U_q(N^+)^{\mathrm{DK}}, M)$$

$$M \longrightarrow C^\cdot(U_q(N^+)^{\mathrm{sml}}, M)$$

$$\mathrm{Whit}_q(\mathrm{Fl}_G) \overset{J_!}{\underset{J_*}{\substack{\longrightarrow\\ \xrightarrow{J_{!*}} \\ \longrightarrow}}} \mathrm{Whit}_q(\mathrm{Gr}_T)$$

$$S^{-,\lambda} \xrightarrow{i^{-,\lambda}} \mathrm{Fl}_G$$

$$\overset{\shortparallel}{N^-(K)\cdot t^\lambda}$$

$$S^{\lambda} \xrightarrow{i^{\lambda}} \mathrm{Fl}_G$$

$$\overset{\shortparallel}{N(K)\cdot t^\lambda}$$

$$H(S^{-,\lambda}, (i^{-,\lambda})^!(\mathcal{F}))$$

$$\overset{\shortparallel}{J_*}$$

$$\cong$$

$$H(\mathrm{Fl}_G,\ i^{-,\lambda}_*(\omega_{S^{-,\lambda}}) \overset{!}{\otimes} \mathcal{F})$$

$$J_!(\mathcal{F})$$

$$\parallel$$

$$H(\mathrm{Fl}_G,\ i_!^{-\lambda}(\omega_{S^{-\lambda}}) \overset{!}{\otimes} \mathcal{F})$$

$$J_{!*}(\mathcal{F})$$

$$\parallel$$

$$H(\mathrm{Fl}_G,\ IC^{\frac{\infty}{2}} \overset{!}{\otimes} \mathcal{F})$$

$J_!^\lambda$ is corepresented by

$$\mathrm{Hom}(M^\lambda, -) \qquad \mathrm{Rep}_q^{\mathrm{mixed}}(G)^{\mathrm{unipotent}}$$

$$\parallel$$

$$\mathrm{ind}^{\mathrm{mixed}}_{U_q(n^+)\otimes\ldots}(k^\lambda)$$

$J_!^\lambda$ is corepresented by in $\mathrm{Whit}_q(\mathrm{Fl}_G)$

$$\parallel$$

$$\mathrm{Av}_!^{N(K),\chi}(j_\lambda) = \mathrm{Av}_!^{N(K),\chi}(\delta_{t^\lambda})$$

λ - dominant

BMW-sheet

$$\hat{j}_\lambda = \begin{cases} j_{\lambda,!} & \lambda\text{-dim} \\ j_{\lambda,*} & \lambda\text{-anti-dim.} \end{cases}$$