Gl oh

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Goal: Formulate some math. conjectures (related to local Langlands for $GL(n)$) and "deduce" them from the formalism that Sam explained last time (Formalism 3d $N=4$ QFT)

Notation: $n \geq 1$

$W_n = \{ \mathcal{E}_1 \to \mathcal{E}_2 \to \dots \to \mathcal{E}_n \}$

$\mathcal{E}_i$ - local system of rank i on $\mathcal{D}^* = \operatorname{Spec} K$

$K = \mathbb{C}((t))$ $\pi_n : W_n \to LS_n(\mathcal{D}^*)$

Conjecture

1. $D\text{-mod}(Gr_{GL(n)})^{Hecke} \simeq \operatorname{IndCoh}(\pi_n^{-1}(\text{triv}))$

2. $\operatorname{Whit}(GL(n, K)) = \operatorname{QCoh}(W_n)$

3. $D\text{-mod}/GL(n,K)) = \operatorname{IndCoh}\left(W_n \underset{LS_n}{\times} W_n \right)$

Close relative of 1:

$S_n = \{ V_1 \to V_2 \to \dots \to V_{n-1} \to \mathbb{C}^n \}$

$\dim V_i = i$

<u>Note:</u> $\pi_n^{-1}(\mathrm{triv}) = \mathrm{Maps}(D^*_{dR}, S_n)$

<u>Conj 1':</u> $\text{D-mod}(Gr_{GL(n)})^{Hecke} \simeq$

$\text{D-mod}(S_n(K)) \qquad T[GL(n)] = T(S_n)$

Back to "physics"

Recall: given a 3d $N=4$ theory T

it has two top. twists (A and B)

$\mathcal{C}_C(T)$ and $\mathcal{C}_H(T)$ categories attached to S^1

Coulomb A — Higgs B

T^* – mirror dual interchanges A and B

<u>Often</u> we can start with some explicit T

and its T^* is also explicit $\rightsquigarrow$ equiv. of cat.

Recall, that given a smooth "affine" stack Y

($Y = Z/G$ Z-smooth aff. G-reductive)

one can construct $T(Y)$ (secretly it should depend only on T^*Y)

$\mathcal{C}_C(T) = \text{D-mod}(Y(K))$

should depend only on T, Y)

$$\mathcal{C}_L(T) = D\text{-mod}(Y(K))$$

$$\mathcal{C}_H(T) = \mathrm{IndCoh}(\mathrm{Maps}(D^*_{dR}, Y))$$

One way to obtain equivalence is to somehow claim that $T(Y)^* \simeq T(Y^*)$ for some Y^*

Also, Sam defined $\forall$ reductive G certain theory $T[G]$ (has G flavour symmetry $\Leftrightarrow$ comes from a boundary cond. for $YM(G)$)

Exercise on Sam's talk:

1) $T[G]^* = T[G^\vee]$

2) Given T - theory with G-symmetry one can form its S-dual $T^\vee$ has G^L-sym.

If T has G-action $\rightsquigarrow$ T/G "gauge G"

ex. if G acts on Y

$T(Y)$ has G-symmetry

$T(Y)/G = T(Y/G)$

$$\underset{\text{S-dual}}{\overset{\shortparallel}{T^\vee}} = (T \times T[G] / \Delta G)^*$$

3) $\mathcal{C}_C(T[G]) = D\text{-mod}(Gr_G)^{\text{Hecke}}$

Given T $\exists$ $\mathcal{C}_{10}$ ... attached to S^1

Given T $\exists\ \mathcal{C}_H, \mathcal{C}_C$ attached to S^1

$S^1 = \partial D$ gives objects $\mathcal{F}_H \in \mathcal{C}_H, \mathcal{F}_C \in \mathcal{C}_C$

$\mathcal{F}_C(T[G]) = R_{G^\vee}$ - D-module which corr. to reg. rep. of $G^\vee$ under geom. Satake

$\mathcal{C}_H(T[G]) = \text{D-mod}(Gr_{G^\vee})^{\text{Hecke}}$

For general reductive G

$T[G]$ is <u>not</u> of the form $T(Y)$ for any Y

If $G = GL(n)$ then it is.

<u>Claim</u> $T[GL(n)] = T(S_n)$

$S_n = \{ (V_1 \to V_2 \to \dots \to V_{n-1} \to \mathbb{C}^n) \}$

$\dim V_i = i$