Derived Geometric Satake with modular coefficients.

G_F, $F = k((t))$, Gr

$D_{G(O)}(Gr) = D({}_{G_O}\backslash G_F/G_O)$

$\cup$

$Perv_{G(O)}(Gr) \underset{MV}{\simeq} Rep(G^\vee)$

Thm (suggested by Drinfeld)

$$D_{G(O)}(Gr) \simeq DCoh^{G^\vee}(\{1\} \underset{\check{\mathfrak{g}}}{\times} \{1\}).$$

(in char 0 follows from Ginzburg (${}_{RB}$ Finkelberg))).

the method use, e.g. weights, doesn't work with coefficients ~~in~~ K - field of char $\ell > 0$ (ℓ is not too small)

Idea comes from R.K. Gordin

$G^\vee/G^\vee$ – stack of L-S parameters.

$\operatorname{End}_{\tilde{D}(\backslash G_F/I^\circ,\psi)}\left(\tilde{D}(I^\circ_{,\psi}\backslash G_F/G_0)\right)$

$\parallel$

Idea of proof.

$$\tilde{D}_{G(0)}(\mathcal{Y}_r) \rightleftarrows \tilde{D}\left({}_{G_0}\backslash G_F/I^\circ,\psi\right) \otimes \tilde{D}\left({}_{I^\circ,\psi}\backslash G_F/G_0\right)$$

Arinkin's talk

$$\tilde{D}\left({}_{I^\circ,\psi}\backslash G_F/I^\circ,\psi\right)_u$$

sizes.

$$D\left({}_{G_0}\backslash G_F/I^\circ,\psi\right) \simeq D(\operatorname{Rep} G^\vee)$$

Gaitsgory Frenkel Vilonen in char 0 "geom Casselman Shalika"

over k of char $l>0$ (some ass-ns) BG Mirković Roder, Riche

$$D\left({}_{I^\circ,\psi}\backslash G_F/I^\circ,\psi\right)$$

I° - radical of Iwahori; ψ - Whittaker character

$\tilde{D}$ - is the large category ~~of~~ constructible?

Rmk. "small" version follows.

Fact $G \curvearrowright X$, $Av_{U,\psi} : D_{B^-}(X) \longrightarrow D_{U,\psi}(X)$
commutes with duality.

Proof that $D\left({}_{I^\circ_-,\psi}\backslash G_F / I^\circ, \psi\right)_u \simeq D\,Coh_{\mathcal{U}}^{G^\vee}(G^\vee)$

$\mathcal{U}$ ← set theoretic support of $\mathcal{U} \subset G^\vee$

full subcat generated by the image of averaging

$Av : D({}_{I_-}\backslash G_F / I_0, \psi) \longrightarrow$

$I, I_- \subset G(O)$

$\mathcal{U}$ - unipotent locus.

Lemma.

$$\underline{D^b_{\mathrm{perf}}\left(\mathrm{Coh}^{G^\vee}_{\mathcal{U}}\left(G^\vee \times_{T^\vee/W} T^\vee\right)\right)} \hookrightarrow D\left(\underline{I}_- \backslash G_F/_T\, I,\psi\right)$$

$$\big\downarrow \; Av_{\underline{I},\psi}$$

$$D\left({}_{\underline{I},\psi}\backslash G_F / I,\psi\right).$$

s.t. the image $=$ $\mathrm{Ker}(Av_{I,\psi})^{\perp}$

$$\| $$

$$\langle \mathcal{L}_w,\ w \notin {}^f W \rangle.$$

$\tilde{G} \longrightarrow \underline{G^\vee \times_{T^\vee/W} T^\vee}$ – unipotently monodromic

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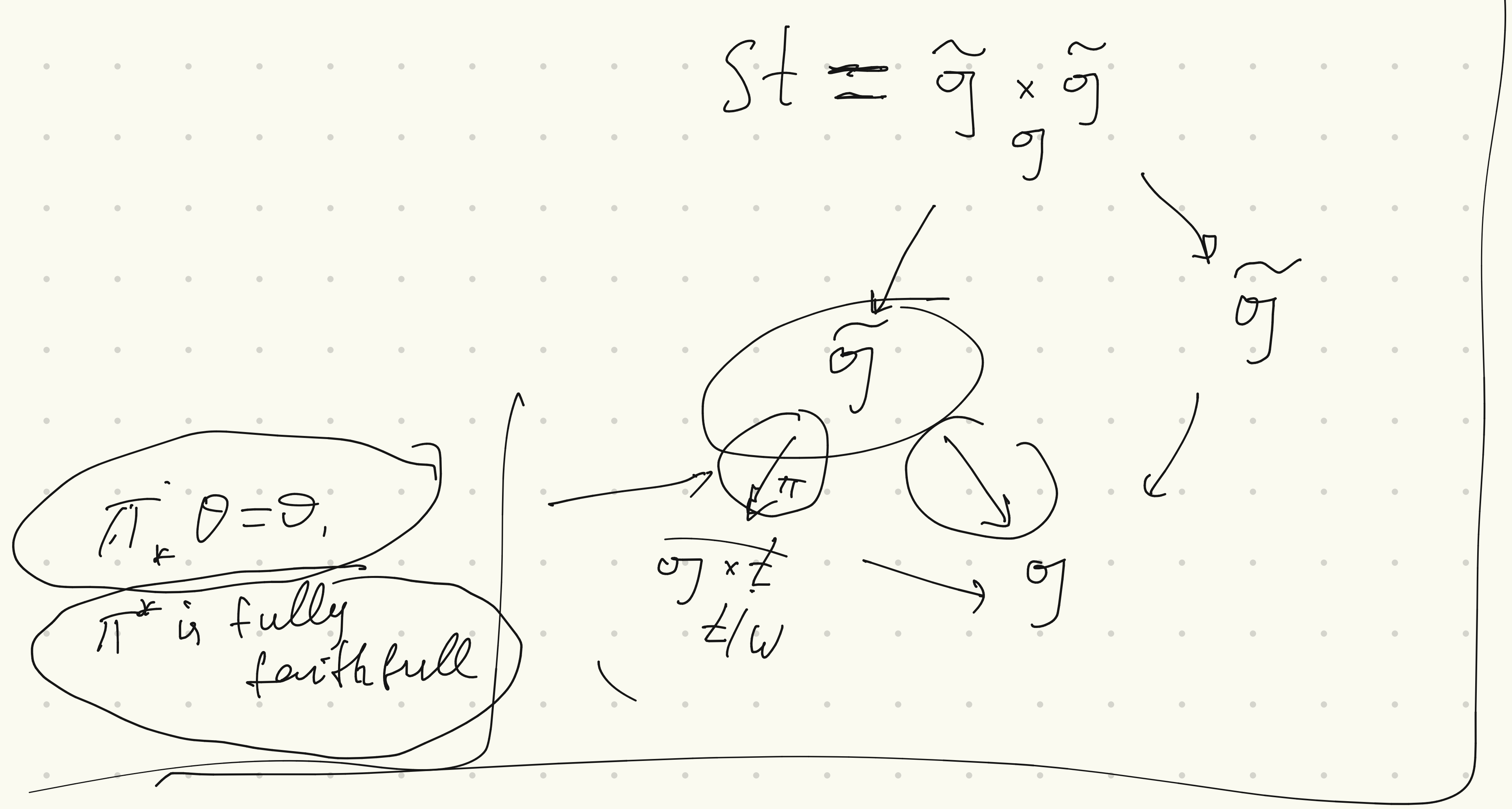
Lemma $\Rightarrow$ the equivalence $D\left({}_{I,\psi}\backslash G_F/I,\psi\right)_u \simeq \underline{D\mathrm{Coh}^{G^\vee}(G^\vee)_u}$.

(partly relies on a result with T. Deshpande)

[MS]

Rmk. passing from $G^\vee \times_{T^\vee/W} T^\vee$ to $G^\vee$ is similar to passing from $D({}_B\backslash G/U)/(\mathrm{Ker}$ Transl to the wall$) \simeq$ sing. block for $\tilde{\mathcal{U}} = \mathcal{U} \otimes_{\mathbb{C}[t/W]} \mathbb{C}[t]$ to $D({}_B\backslash G/_{U,\psi})$ '' sing. block



building a functor from $\mathrm{Coh}^{G^\vee}(G^\vee)$ ~~from~~ functor from $\mathrm{Rep}(G^\vee)$ use monodromy aut-m of $\hat{Z}$ + Tannakian f-m.

Proof of Lemma: use central functors

$$\begin{array}{ccc} \hat{Z} : \mathrm{Rep}(G^\vee) & \longrightarrow & D({}_{I_-}\backslash G / I_-) \\ \downarrow & \nearrow & \downarrow \\ \mathrm{Coh}^{G^\vee}(G^\vee \times_{T^\vee/W} T^\vee) & \longrightarrow & D({}_{I_-}\backslash G / I, \psi) \end{array}$$

$V \mapsto V \otimes \mathcal{O}$

Claim $\mathrm{Rep}(G^\vee) \longrightarrow \mathrm{Perv}({}_{I_-}\backslash G / I, \psi)$ sends tilting to tilting.

see B Riche, Rider.

Th

This + description of the quotient category

$\mathrm{Perv}(I \backslash G_F / I, \psi) \big/ \langle L_w,\ \ell(w)>0 \rangle$ $\Rightarrow$ full faithfulness

$\mathrm{Perv}(I \backslash G_F / I)$

in (BRR) such q-nt of $\mathrm{Perv}(I \backslash G_F / I) \simeq \mathrm{Rep}(Z(u)_{G^\vee})$

u-regular.

Lemma $\Rightarrow$ eq-nce.

Claim $G \circlearrowright X$. $D(U,\psi \backslash X) \supset D(U,\psi \backslash X)_u$ span of the image

$$\uparrow AV$$

$K[T^\vee] \circlearrowright$ monodromy $D(B_- \backslash X)$

$$D(B_- \backslash X) \supset \mathrm{Ker}(AV)^\perp \simeq D(U,\psi \backslash X) \otimes_{K[T^\vee/W]} K[T^\vee] \hookleftarrow W.$$

$$\frac{D(B_- \backslash X)}{\mathrm{Ker}(Av)} \circlearrowleft W.$$

$$D(B_- \backslash G/U,\psi) \otimes D(U,\psi \backslash G/U,\psi)$$

$$D(U,\psi \backslash X) \hookrightarrow D(B_- \backslash X)$$

by Arinkin

$$\cancel{= D(T)}$$

$$D(U_- \backslash G/U,\psi) \simeq D(T)$$

$$D(B_- \backslash G/U,\psi) \simeq \mathrm{Coh}_1(T^\vee)$$

$$D(U,\psi \backslash G/U,\psi)_u \simeq \mathrm{Coh}_1(\check{T}/W)$$

So

$$\widetilde{B}\left({}_{B_-}\backslash X\right) \simeq \widetilde{D}\left({}_{U,\psi}\backslash X\right)_u \underset{k[T^\vee/W]}{\otimes} k[T^\vee].$$

In our case want to describe

$D\left({}_{U,\psi}\backslash X\right)_u$, first get $D\left({}_{B_-}\backslash X\right) =$

$= ? \underset{k[T^\vee/W]}{\otimes} k[T^\vee]$.

Check W actions match + descend.

$$X \not\leftarrow {}_{G_+(0)}\backslash G_F / I,\psi$$

$$G_+(0) = \mathrm{Ker}\,(G(0) \to G)$$

Question: Can one run the construction of Z directly for $D\left({}_{I,\psi}\backslash G_F / I,\psi\right)_u$.