Fact (Bogarchenko–Drinfeld)

G = group

$\langle Shv(G), \star \rangle$ is rigid   constructible / holonomic D-modules.

if G is projective.

Example: D-modules $G = \mathbb{G}_m$

$G = \mathbb{A}^1$ $\langle Shv(\mathbb{G}_a), \star \rangle \simeq \langle D\text{-mod}(\mathbb{A}^1), \otimes \rangle$

G = Elliptic curve $\langle Shv(G), \star \rangle = \langle \mathrm{Perf}(\text{smooth}), \otimes \rangle$.

Observation $H \subset G$ $\langle Shv(H\backslash G/H), \star \rangle$ rigid

if G/H is projective

Observation: $G \supset \tilde{H} \supset H$

$G/\tilde{H}$ is projective

$\tilde{H}/H$ is a torus.

$\langle Shv(K\backslash G/H)^{\tilde{H}/H\text{-monodromic}}, \# \rangle$-rigid

Formalism

X = scheme / stack $\quad Shv(X)$ "Ind(constructible sheaves)"

Functors $f_*, f^!, \otimes$.

G = group

G-space $\quad$ G-Sch. 2-category

$X, Y \in G$-Sch

$$Mor(X,Y) = Shv(X \times Y / G).$$

composition = convolution

Duality:

~~① On objects.~~

② Duality on 1-morphisms.

does $f \in Mor(X,Y)$ have a right adjoint $f^R \in Mor(Y,X)$

schematic over BG.

Fact If $X \in G/H$-projective.

Then any $f \in \mathrm{Mor}(X,Y)^c$ has a right adj. $f^R \in \mathrm{Mor}(Y,X)^c$

any $f \in \mathrm{Mor}(Y,X)^c$ has $f^L \in \mathrm{Mor}(X,Y)^c$,

Theorem For any Y, Z

$Y \to X \to Z$

$$\mathrm{Mor}(X,Z) \underset{\mathrm{End}(X)}{\otimes} \mathrm{Mor}(Y,X) \xrightarrow{\star} \mathrm{Mor}(Y,Z).$$

$\star$ is fully faithful.