

Nodal degenerations in Betti Geometric Langlands

Joint with Zhiwei Yun & Penghui Li Ben-Zvi
(1606.08523)

Plan Talk 1 (DN) Overview, spectral motivations,
automorphic assertions, questions

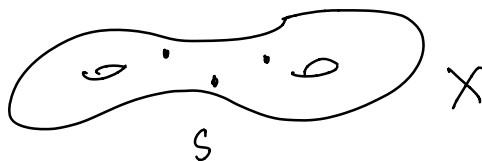
Talk 2 (ZY) Geometry of automorphic assertions
(2105.12318)

Talk 3 ... (PL) Functions on commuting stacks
(22...?)

① Setup Everything / $\mathbb{C}$, classical topology ...

$$\begin{array}{ccc} G \hookrightarrow B \twoheadrightarrow H & & G^v \hookrightarrow B^v \twoheadrightarrow H^v \\ \uparrow u & & \uparrow u^v \end{array}$$

(X, S) X sm proj curve, $S \subset X$ fin. marked points



A-side, Geometric Side

Automorphic side $\text{Bun}_{G,u}(X,S) = G\text{-bundles on } X$
with $u\text{-red. over } S$

nilpotent cone $\mathcal{N} \subset T^* \text{Bun}_{G,u}(X,S)$

- Hitchin map $T^* \text{Bun}_{G,u}(X,S) \xrightarrow{h} \mathcal{A}_{G,u}(X,S)$
 $\mathcal{N} = h^{-1}(0)$

- Induction map $\text{Bun}_{B,u}(X,S) \xrightarrow{p} \text{Bun}_{G,u}(X,S)$
 $\mathcal{N} = \Lambda_p$ cone of singl of
map p .

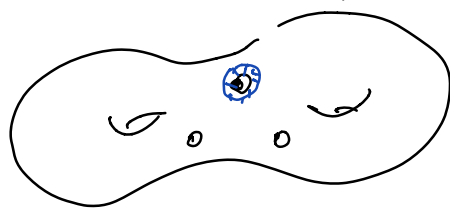
Automorphic category: $\mathcal{A}(X,S) = \text{Sh}_{\mathcal{N}}(\text{Bun}_{G,u}(X,S))$
 $\uparrow$
Kashiwara-Schapira

B-side, Algebraic side

Spectral side

(Betti)

$\text{Loc}_{G^v, B^v}(X,S) = G^v\text{-local systems on } X \setminus S$
with $B^v\text{-red. near } S$



$$= \text{Loc}_{G^v}(X \setminus S) \times \left(\frac{B^v}{B^v} \right)^S$$

$$\left(\frac{G^v}{G^v} \right)^S$$



Spectral category $\mathcal{B}(X, S) = \text{Ind Coh}_{N^v}(\text{Loc}_{G^v, B^v}(X, S))$
 $\uparrow$
 Arinkin - Gaitsgory

Betti Geom Langlands Conj: $A(X, S) \simeq \mathcal{B}(X, S)$

Some compatibilities

1) Parabolic induction

(Eisenstein series:

$$\left\{ \text{Bun}_{H,1}(X, S) \leftarrow \text{Bun}_{B,u}(X, S) \rightarrow \text{Bun}_{G,u}(X, S) \right\}$$

2) Normalization

Spectral side: structure sheaf
 corepresentations global sections

$$\Gamma(\text{Loc}_{G^v, B^v}(X, S), -) \simeq \text{Hom}(\mathcal{O}, -)$$

Aut. side: Whittaker functional

Rank: $\text{Hom}(\text{Wh}, -)$
 $\uparrow$
 N -sing supp corep. obj.

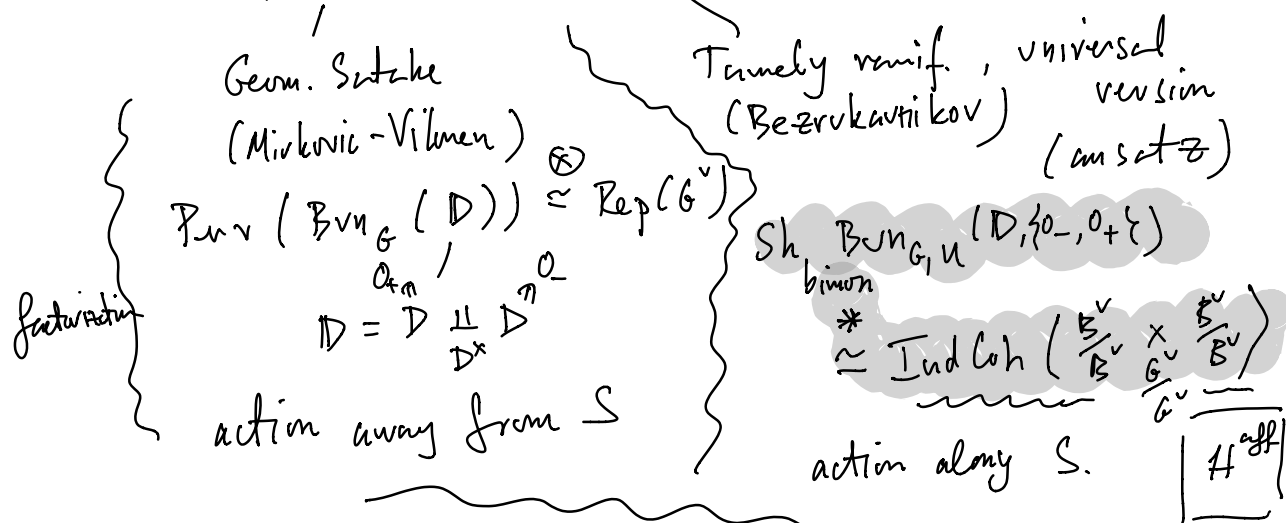
" N -Taylor

microstalk of shvs
 at particular corep.

Whittaker diagram

$$\left\{ A' \leftarrow \text{Bun}_{u^-, u}^{v(w)}(X, S) \xrightarrow{w(X, S)} \text{Bun}_{G, u}(X, S) \right\}$$

3) Spherical & affine Hecke actions



Commutative part $\leadsto$ Spectral action (N-Yun)

$$\text{QCoh}(\text{Loc}_{G^\vee, \text{B}^\vee}(X, S)) \xrightarrow{\text{Hecke}} A(X, S)$$

$$\xrightarrow{\otimes} B(X, S)$$

(II) Dependence on (X, S) ? Spectral results & observations

Observation $B(X, S) = \text{Ind Coh}_{\mathcal{N}^\vee}(\text{Loc}_{G^\vee, \text{B}^\vee}(X, S))$

loc. const $(X, S) \in \mathcal{M}_{g,n}$

Conj. $A(X, S)$ loc const in (X, S)

Perhaps more scientific approach (?) may be discussed further by Zhiwei

$$\mathrm{Bun}_{G,u}(g,n) \xleftarrow{p} \mathrm{Bun}_{B,u}(g,n)$$

$\pi \downarrow$

$\mathcal{M}_{g,n}$

Form "universal" nilp cone

$$\tilde{\mathcal{N}}_{g,n} := \Lambda_p \quad \text{"sing codim of } p \text{"}$$

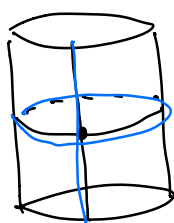
Lift of nilp cone

$$\mathcal{N}_{g,n} \subset T^*\pi$$

$$\tilde{\mathcal{N}}_{g,n} \subset T^*\mathrm{Bun}_{G,u}(g,n)$$

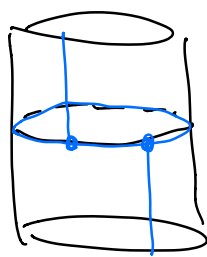
Conj $Sh_{\tilde{\mathcal{N}}_{g,n}}(\mathrm{Bun}_{G,u}(g,n))$ loc const in $\mathcal{M}_{g,n}$
with stalk $Sh_{\mathcal{N}}(\mathrm{Bun}_{G,u}(X,S))$

Cartoon



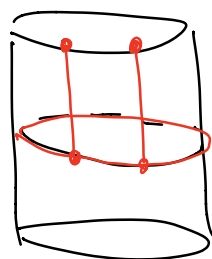
T^*S^1

Λ_0

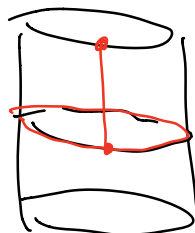


Λ_ϵ

$$Sh_{\Lambda_0}(S^1) \simeq Sh_{\Lambda_\epsilon}(S^1)$$



Λ_0



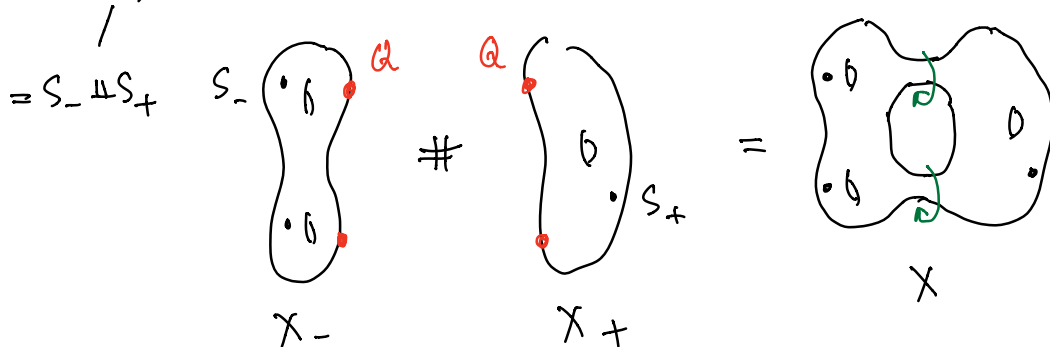
Λ_ϵ

Ex Conj holds $\mathcal{M}_{1,0}$ ie for genus 1-curves
($L_i - N$)

$$ms \quad SL_2(\mathbb{Z}) \subset A(E) \quad , \quad E \in \mathcal{M}_{1,0}$$

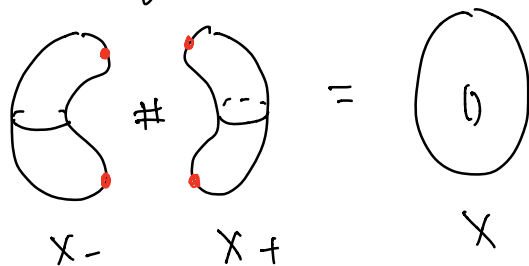
Spectral Gluing (Ben-Zvi - N - Preygel ...)

$$(X, S) = (X_-, S_- \sqcup Q) \# (X_+, S_+ \sqcup Q)$$



Thm $B(X, S) = B(X_-, S_- \sqcup Q) \otimes_{(H^{aff})^Q} B(X_+, S_+ \sqcup Q)$

Ex $X = \mathbb{P}^1$ genus one



$$hh(H^{aff}) := \frac{H^{aff} \otimes H^{aff}}{H^{aff} \otimes (H^{aff})^{\otimes 2}} \simeq \mathcal{B}(E) = \text{IndCoh}_{\mathcal{N}^v}(Loc_{G^v}(E))$$

cocenter
of H^{aff} as monoidal cat.

[Runk Germin Stefanich $\rightsquigarrow$ extended 2d TFT
on spectral side
 $pt \mapsto "3\text{IndCoh}_{\mathcal{N}^v}(BG^v)"$]

(III) To be continued...