

$$\begin{array}{ccc}
\text{Whit}(Gr_G)_{\text{Ran}} & \xrightarrow{\sim} & \text{Rep}(\check{G})_{\text{Ran}} \\
\text{cost}_G \uparrow & & \uparrow \Gamma_{\check{G}}^{\text{spec}} \\
D(\text{Bun}_G) & \xrightarrow{LL_G} & \text{IndCoh}_{\text{Nilp}}(LS_{\check{G}}^{\vee}) \\
\text{Loc}_G \uparrow & & \uparrow \text{Poinc}_{\check{G}}^{\text{spec}} \\
KL(G)_{\text{crit}, \text{Ran}} \simeq \text{IndCoh}^* (\text{Op}(D^{\alpha}))_{\text{Ran}} & & \uparrow \text{non-tree}
\end{array}$$

$$D(\text{Bun}_G)^{\text{irred}} := D(\text{Bun}_G) \otimes_{\text{QCoh}(LS_{\check{G}}^{\vee})} \text{QCoh}(LS_{\check{G}}^{\vee, \text{irred}})$$

$$D(\text{Bun}_G)^{\text{irred}} \longrightarrow \text{Ind Coh}_{\text{nilp}} \left(\text{LS}_{\mathbb{C}}^{\text{irred}} \right)_{\text{SI}}$$

$$\text{QCoh} \left(\text{LS}_{\mathbb{C}}^{\text{irred}} \right)$$

Lemmas $D(\text{Bun}_G)^{\text{irred}} \subseteq D(\text{Bun}_G)_{\text{cusp}}$

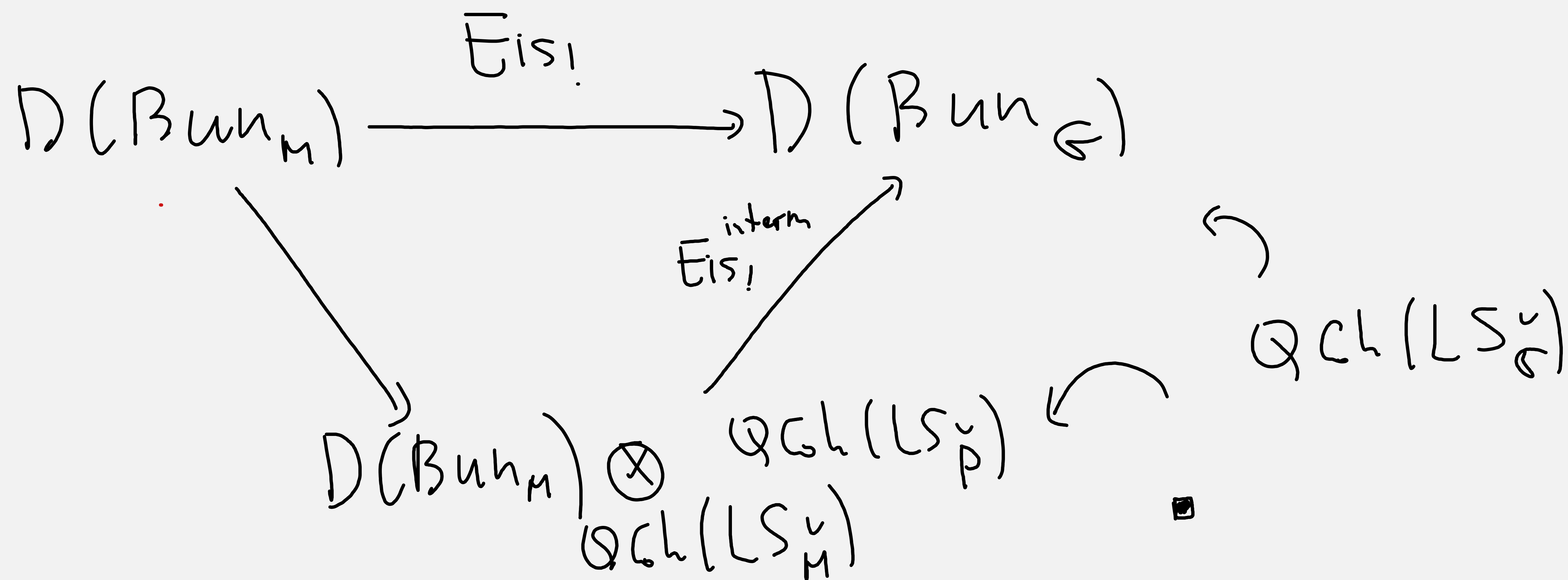
$$D(\text{Bun}_G)_{\text{cusp}} := \left(D(\text{Bun}_G)_{\text{Eis}} \right)^{\perp}$$

Proof of Lemma

Need to show:

$$D(\text{Bun}_G)_{E_{is}} \otimes_{\mathbb{Q}Ch(LS_{\check{G}}^{\text{red}})} = 0$$

\dots $\mathbb{Q}Ch(LS_{\check{G}}^{\vee})$

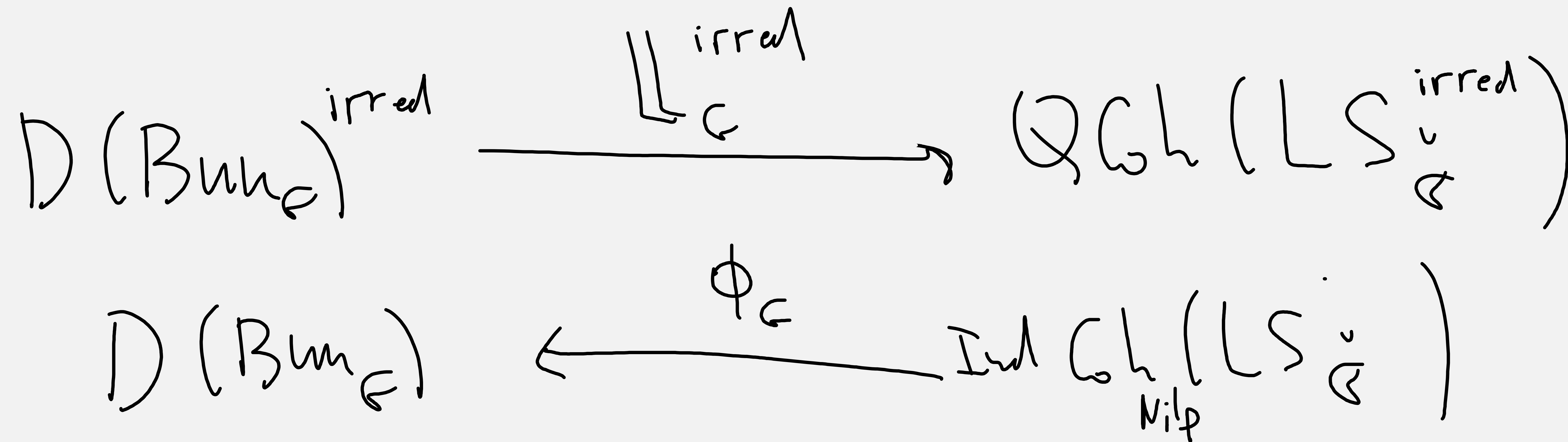


Remark

$$D(\text{Bun}_G)^{\text{irred}} \subseteq D(\text{Bun}_G)_{\text{cusp.}}$$

In fact, this is an equality.

Cor $D(\text{Bun}_G)^{\text{irred}} \subseteq D(\text{Bun}_G)_{\text{temp.}}$



Thm $\mathcal{L}_{\mathbb{G}}^{\text{irred}}$ is both a left and a right
adjoint of $\Phi_{\mathbb{G}}^{\text{irred}}$.

$$V \cdot 0 / \mathbb{G}_m \xrightarrow{j} V / \mathbb{G}_m$$

$$D(\text{Bun}_{\mathbb{G}})^{\vee} =: D(\text{Bun}_{\mathbb{G}})_{\infty}$$

Recall that $D(\text{Bun}_{\mathbb{G}})$ is compactly generated

$$D(\text{Bun}_{\mathbb{G}})_{\infty} := \text{Ind} \left(\begin{array}{c} \text{subset generated} \\ \text{by} \\ j_* (\mathcal{F}_U) \end{array} \right)$$

$$U \xrightarrow{j} \text{Bun}_{\mathbb{G}}$$

$j_! (\mathcal{F}_U), \mathcal{F}_U \in D(U)^c$

$$PsId_* : D(\text{Bun}_G)_{co} \longrightarrow D(\text{Bun}_G)$$

$$\uparrow$$

$$PsId_! : D(\text{Bun}_G)_{co} \xrightarrow{\sim} D(\text{Bun}_G)$$

$$PsId_* \iff \Delta_*(\omega_{\text{Bun}_G}) \in D(\text{Bun}_G) \otimes D(\text{Bun}_G) \simeq D(\text{Bun}_G * \text{Bun}_G)$$

$$PsId_! \iff \Delta_!(\omega_{\text{Bun}_G})$$

$$PsId_* \circ (PsId_!)^{-1} : D(\text{Bun}_G) \longrightarrow D(\text{Bun}_G)$$

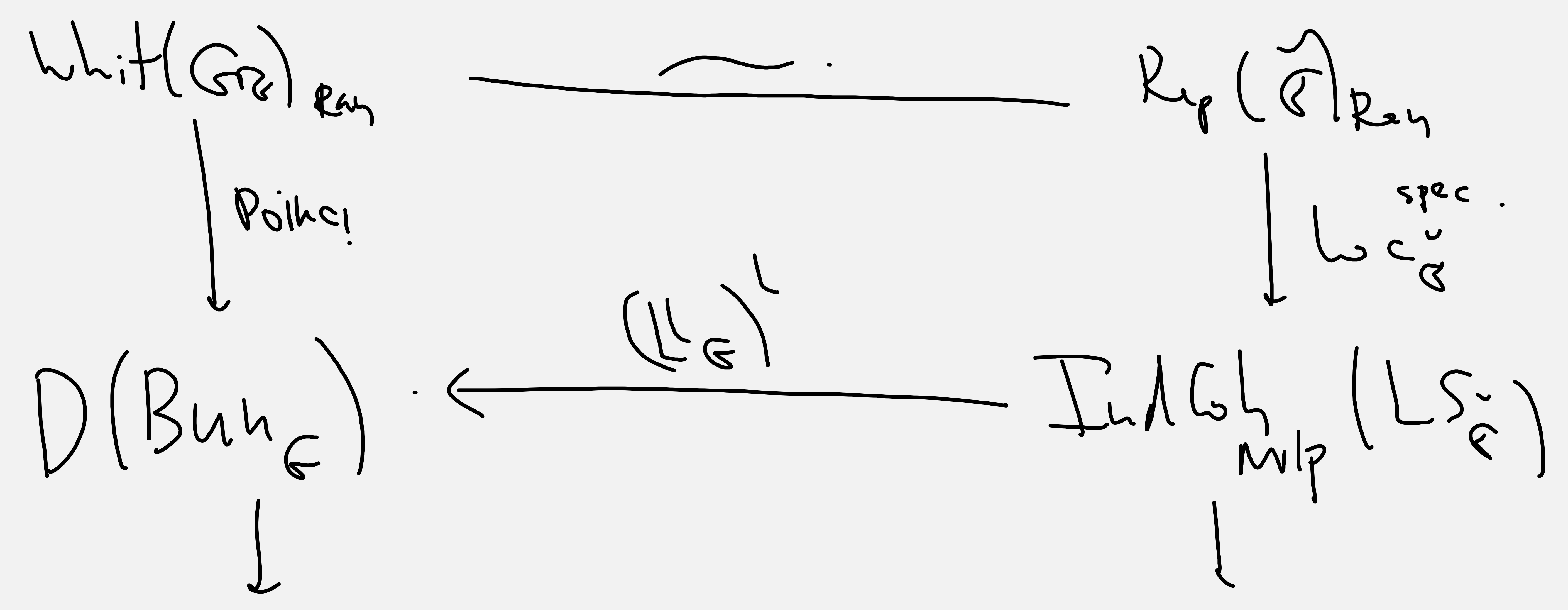
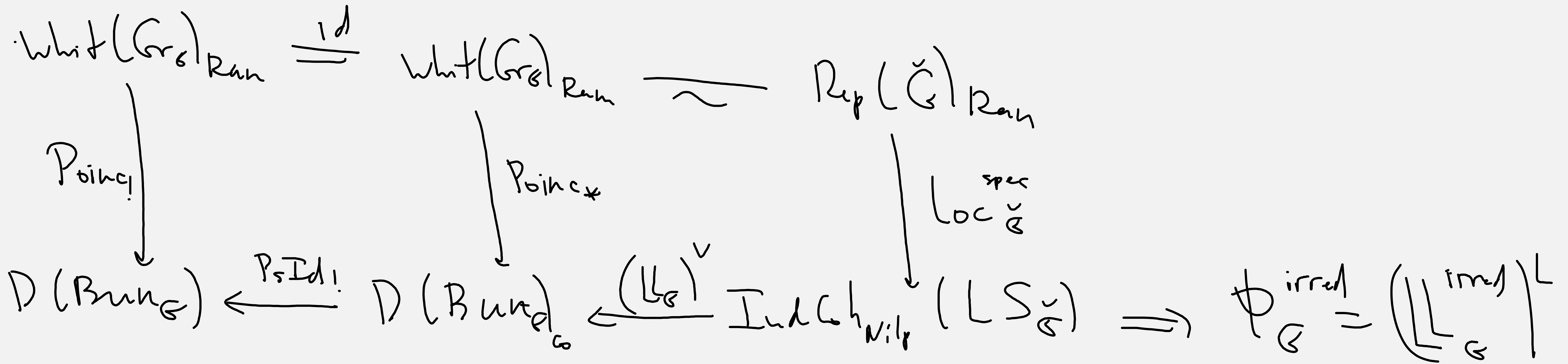
$$\begin{array}{ccc} \text{Ind}_{\text{Nilp}}^G h_{\text{LS}}^{\vee} & \longrightarrow & \text{Ind}_{\text{Nilp}}^G h_{\text{LS}}^{\vee} \\ \downarrow & & \downarrow \\ \mathcal{O}_{\text{GL}}(\text{LS}) & \xrightarrow{\text{st}} & \mathcal{O}_{\text{GL}}(\text{LS}) \end{array}$$

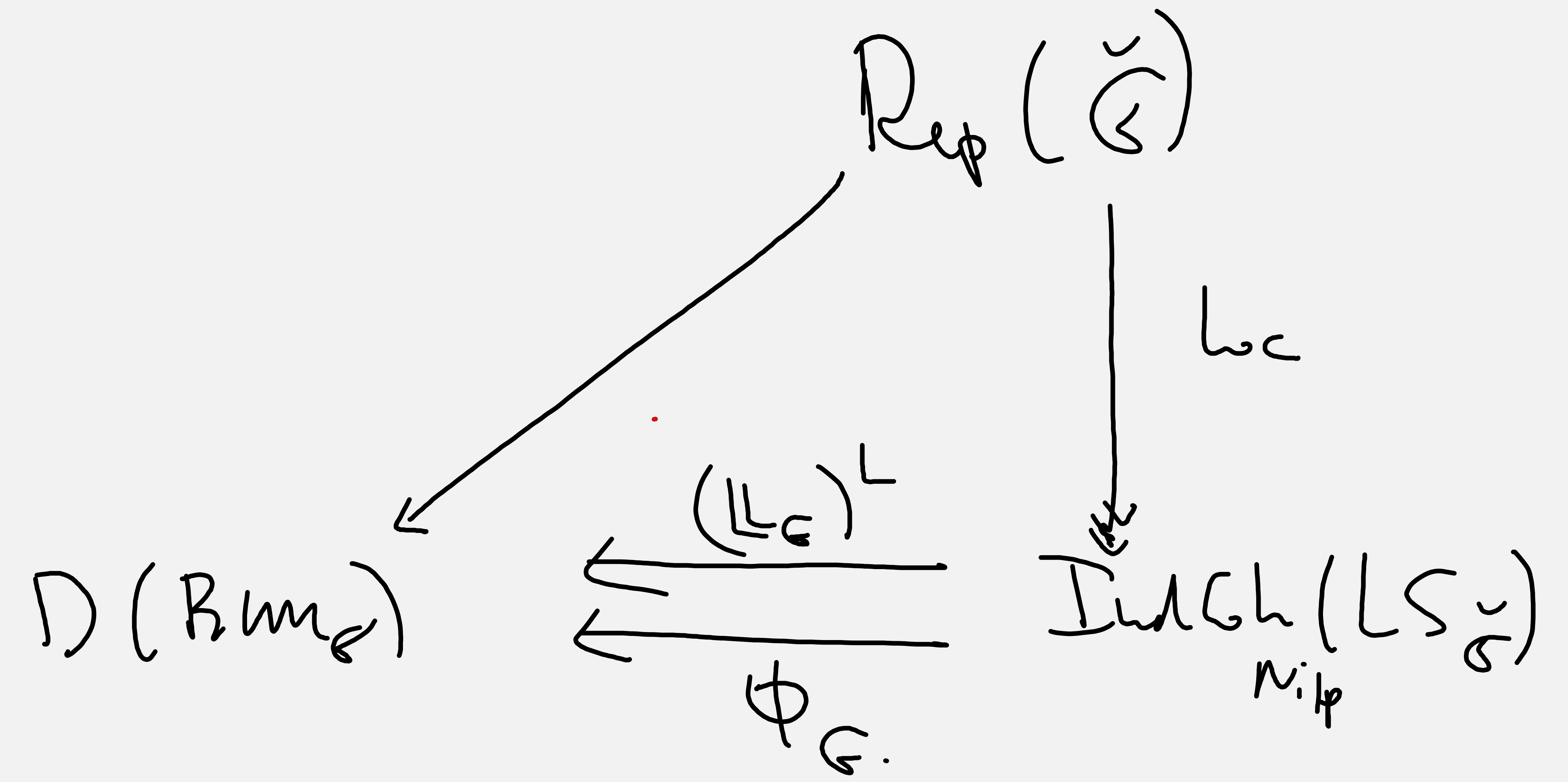
$$D(\text{Bun}_G) \xleftarrow{P \circ \text{Id}_!} D(\text{Bun}_G)_\omega \xleftarrow{\mathbb{L}_G^\vee} \text{Ind}_{\text{Nil}_p} \mathcal{G}_h (LS_{\mathbb{G}})$$

This is ϕ_G .

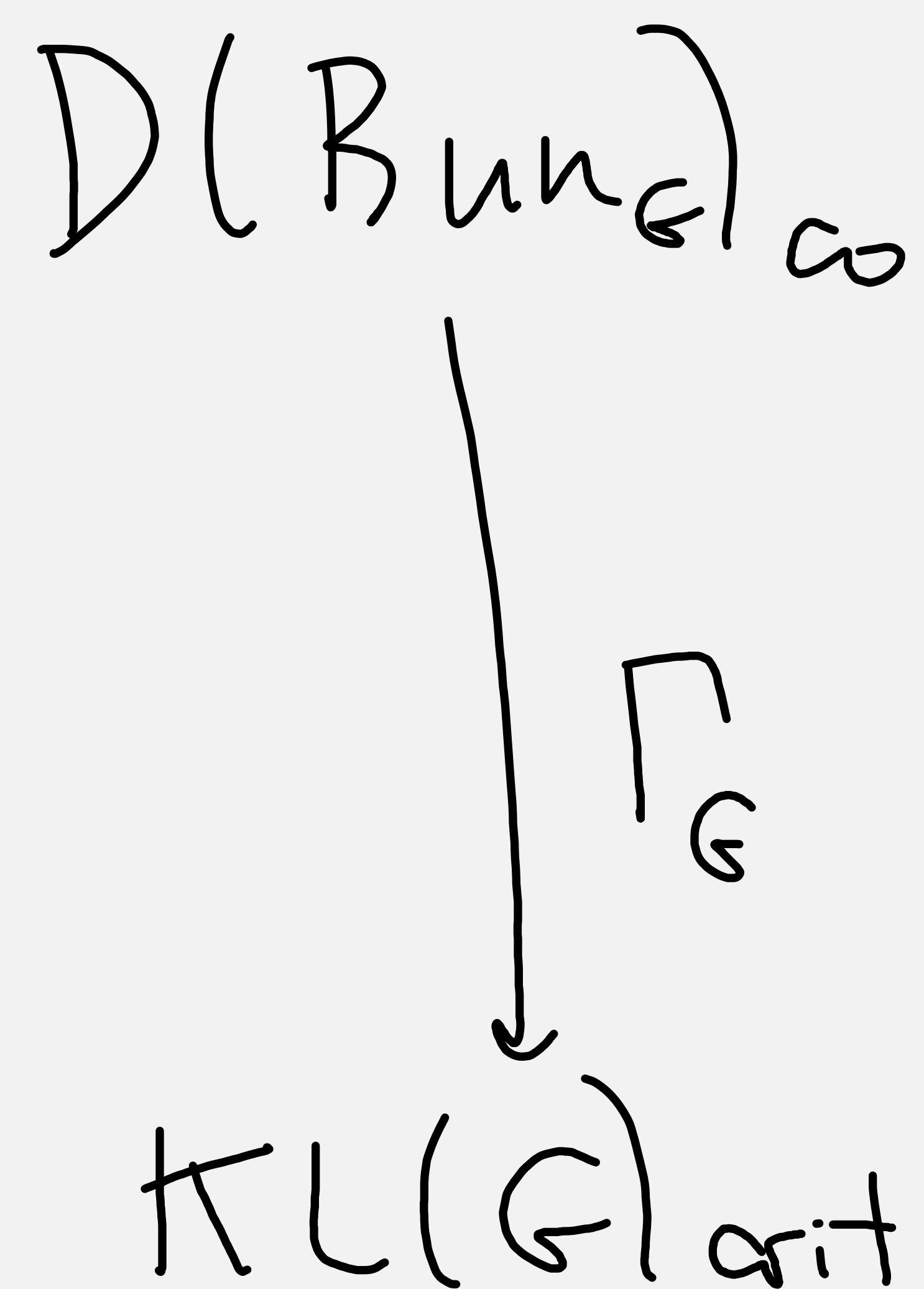
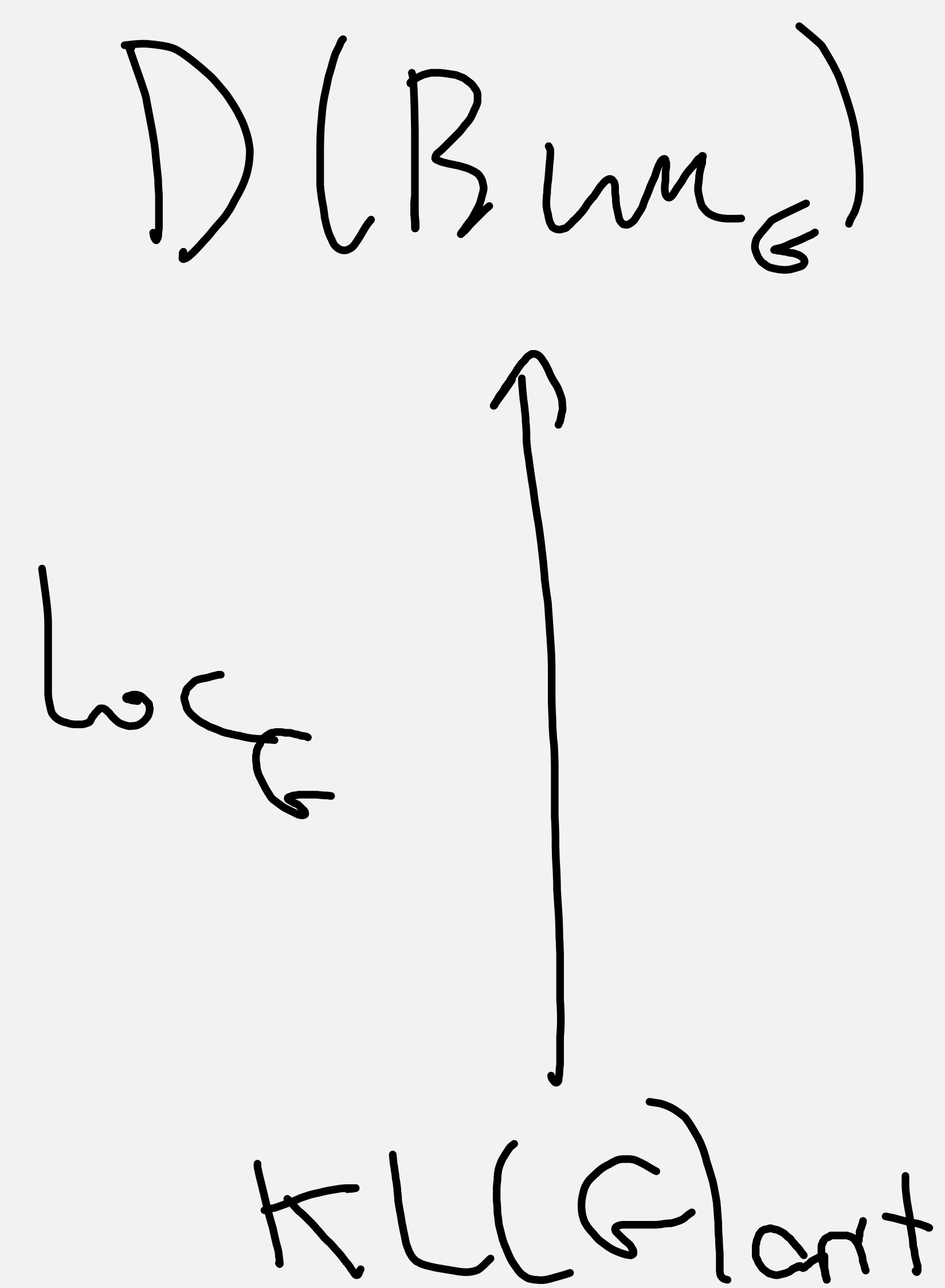
Thm 1 ϕ_G^{irred} is the left adjoint of $\mathbb{L}_G^{\text{irred}}$

Thm 2 ϕ_G^{irred} is the right adjoint of $\mathbb{L}_G^{\text{irred}}$.





Thm 2 L_G^{irred} is the left adjoint of Φ_G^{irred} .



Let $U \xrightarrow{j} \text{Bun}_G$
be g.c.

Lemma A

$(j^* \circ \text{Loc}_G; \Gamma_G \circ j_*$
are adjoint.

$$\begin{array}{c} \mathcal{Q} \text{ Coh} (LS_{\mathbb{C}}^{\text{irred}}) \\ \uparrow j^* \\ \text{Ind Coh}_{\text{Nilp}} (LS_{\mathbb{C}}) \end{array}$$

$$\begin{array}{c} \text{Poinc}_{\mathbb{C}}^{\text{spec}} \\ \uparrow \\ \text{Ind Coh}^* (\mathcal{O}_{\mathbb{P}_{\mathbb{C}}^{\text{mult}}}(D^*)) \end{array}$$

Lemma B $(j^*, \text{Poinc}_{\mathbb{C}}^{\text{spec}}, \text{coeff}_{\mathbb{C}}^{\text{spec}} \cdot j^*)$

$$\begin{array}{c} \mathcal{Q} \text{ Coh} (LS_{\mathbb{C}}^{\text{irred}}) \\ \downarrow j_* \\ \text{Ind Coh}_{\text{Nilp}} (LS_{\mathbb{C}}) \end{array}$$

$$\begin{array}{c} \text{coeff}_{\mathbb{C}}^{\text{spec}} \\ \downarrow \\ \text{Ind Coh}^* (\mathcal{O}_{\mathbb{P}_{\mathbb{C}}^{\text{mult}}}(D^*)) \end{array}$$

Lemma A + Lemma B \Rightarrow Thm 2

$$\begin{array}{ccc}
 & & \text{LS}_{\mathbb{Q}}^{\vee}(x) \\
 & & \uparrow p \\
 \mathcal{O}_{P, \mathbb{Q}}^{\text{mult-free}}(D^x) & \xleftarrow{i} & \mathcal{O}_{P, \mathbb{Q}}^{\text{mult-free}}(X-x)
 \end{array}$$

$$p_{\text{line}}^{\text{spec}} = p_* \circ i^!$$

$$\text{coeff}^{\text{spec}} = i_* \circ p^!$$

$$(\text{coeff}^{\text{spec}})^L = (p^!)^L \circ i^*$$

Want: \bullet p_* is $(p^!)^L$

$$\bullet i^! \cong i^* [\]$$

game with determinant gerbes.

Possibility for next time:

• Lemma $A + B \Rightarrow$ Thm 2.

• The shape of $L_G^{\text{irred}} \cdot \phi_G^{\text{irred}}$ (is an étale algebra).

