

1. Construction of the functor \mathbb{L}_G .
 2. Localization from Kac-Moody modules
 3. Spectral decomposition theorem
 4. Localization and CT functors
-

$$\text{Rep}(\check{G})_{\text{ren}} \curvearrowright D(\text{Bun}_G)$$

$$\text{Loc}_G \downarrow \uparrow \Gamma_{\check{G}, \infty}$$

$$\mathcal{O}Gh(LS_{\check{G}})$$

$$\text{Loc}_G(V_x) = E_{V_x}(V)$$

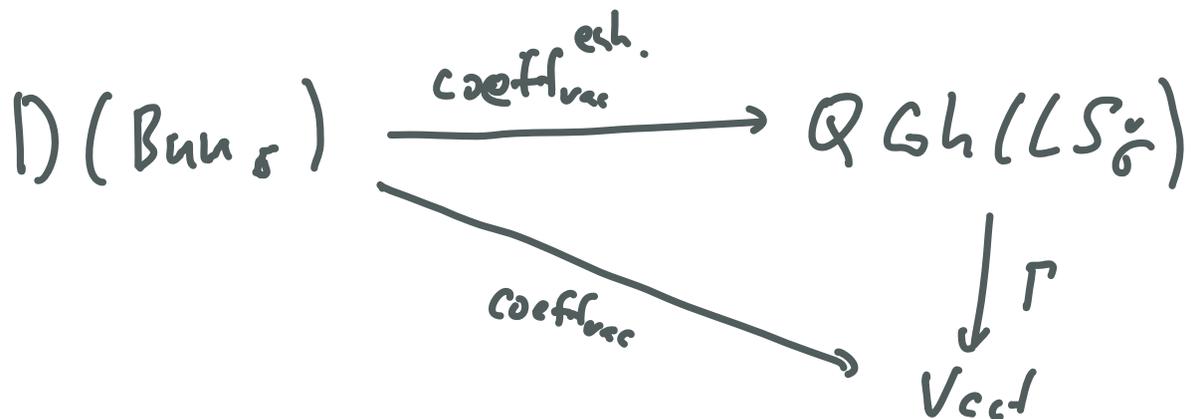
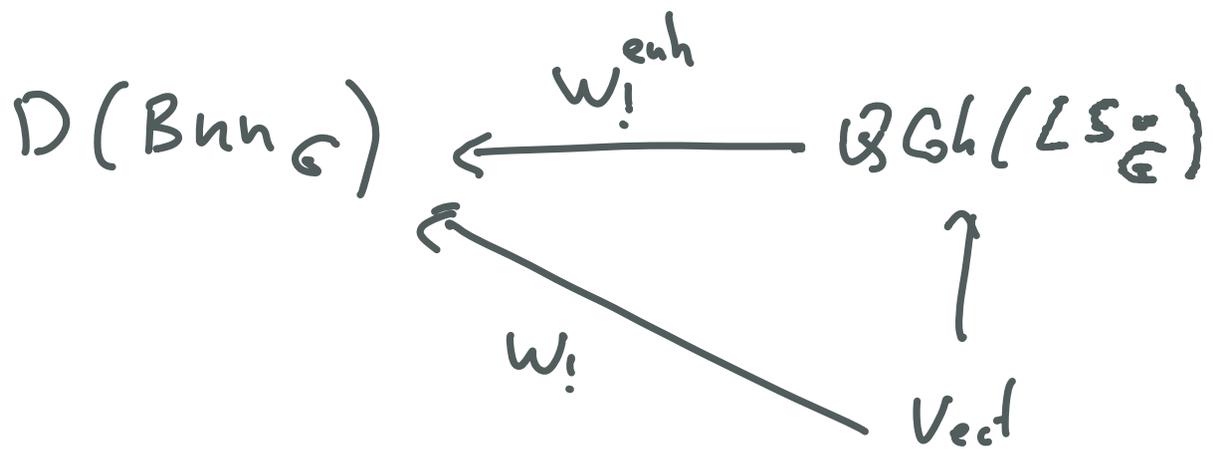
$$x \in X$$

$$V \in \text{Rep}(\check{G})$$

$$E_{V_x}: LS_{\check{G}} \rightarrow \mathcal{P}L/\check{G}$$

Prop $\Gamma_{\check{G}, \infty}$ is fully faithful.

Thm The Hecke action of $\text{Rep}(\check{G})_{\text{Res}}$ on $D(\text{Bun}_G)$ factors through $\text{QCoh}(LS_{\check{G}})$.



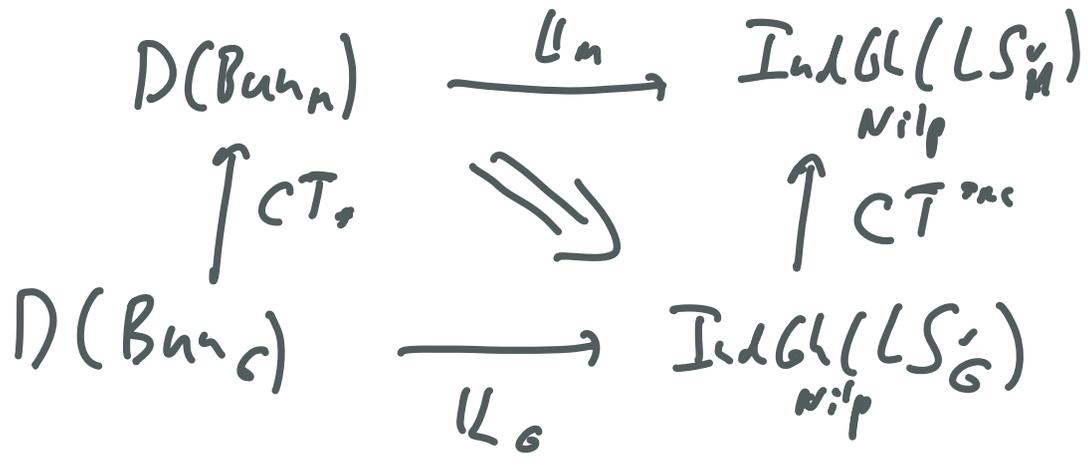
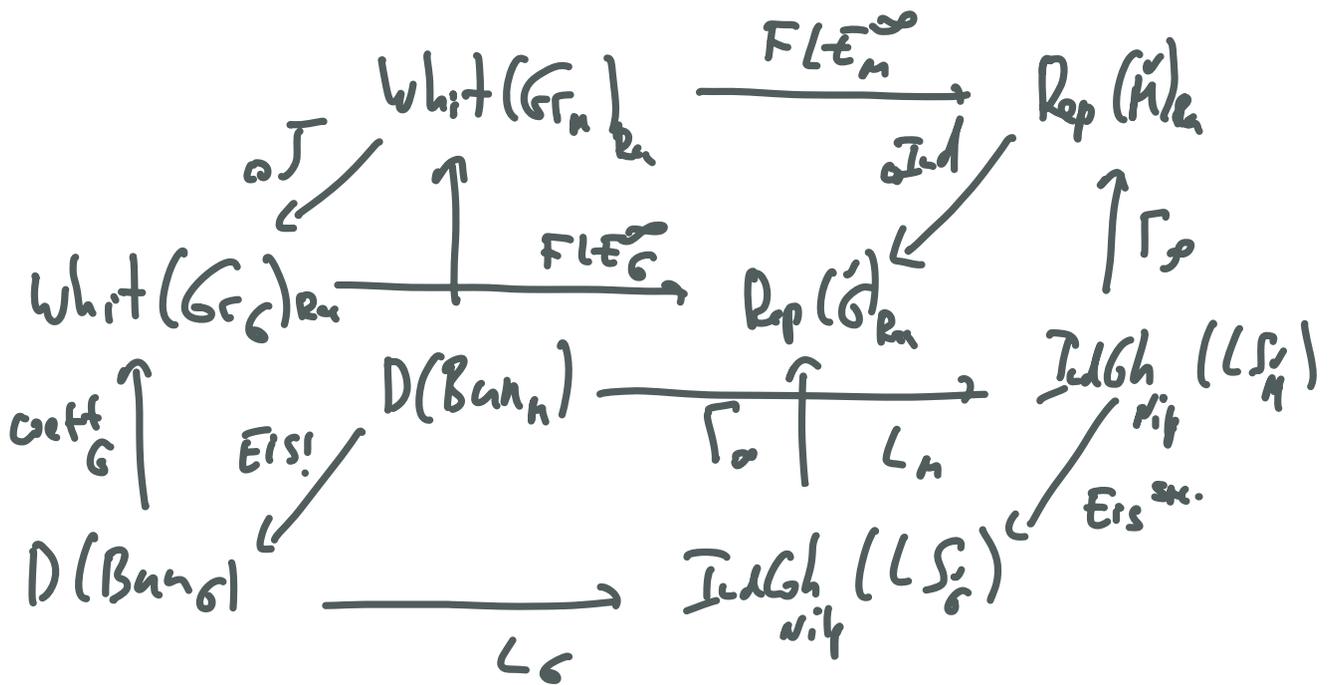
Previous talks:

$$\begin{array}{ccc}
 & \xrightarrow{\text{coeff}^{\text{al}}} & \mathcal{Q}Gh(LS_{\check{G}}) \\
 D(\text{Bun}_G) & \xrightarrow{\mathbb{L}_G} & \text{IndGh}_{\text{nilp}}(LS_{\check{G}})
 \end{array}$$

$$\begin{array}{ccc}
 \text{Whit}(Gr)_{\text{ren}} & \xrightarrow{\text{FLE}^{\infty}} & \text{Rep}(\check{G})_{\text{ren}} \\
 \uparrow \text{coeff} & & \uparrow \Gamma_{\check{G}, \infty} \\
 D(\text{Bun}_G) & \xrightarrow{\mathbb{L}_G} & \text{IndGh}_{\text{nilp}}(LS_{\check{G}})
 \end{array}$$

• \mathbb{L}_G is compatible with derived Satake.

$$\begin{array}{ccc}
 D(\text{Bun}_M) & \xrightarrow{\mathbb{L}_M} & \text{IndGh}_{\text{nilp}}(LS_M) \\
 \downarrow \text{Eis}_! & & \downarrow \text{Eis}^{\text{soc}} \\
 D(\text{Bun}_G) & \xrightarrow{\mathbb{L}_G} & \text{IndGh}_{\text{nilp}}(LS_{\check{G}})
 \end{array}$$



Thm This natural transformation is an iso.

$$\text{Rep}(G) \xrightarrow{\text{Loc}_G} \mathcal{O}GL(LG)$$



$$\begin{array}{ccc}
 \mathrm{KL}(G)_{\mathrm{crit}} & \xrightarrow{\mathrm{Loc}_{G, \mathrm{crit}}} & \mathrm{D}_{\mathrm{und}}(\mathrm{Bun}_G)_{\mathrm{crit}} \\
 & & \parallel \\
 & & \mathrm{D}(\overline{\mathrm{Bun}}_G)
 \end{array}$$

$$\mathrm{KL}(G)_{\mathrm{crit}} = (\hat{a}_{G, \mathrm{crit}}, G(0))$$

$$\begin{array}{ccc}
 \mathrm{Rep}(G(0)) & \xrightarrow{\quad} & \mathrm{QSh}(\mathrm{Bun}_G) \\
 \mathrm{ind} \downarrow & & \downarrow \mathrm{ind} \\
 \mathrm{KL}(G)_{\mathrm{crit}} & \xrightarrow{\mathrm{Loc}_{G, \mathrm{crit}}} & \mathrm{D}(\mathrm{Bun}_G)
 \end{array}$$

$$\mathrm{Bun}_G \xrightarrow{\mathrm{Ev}_x} \mathbb{P}^1 / G(0)$$

• Compatible w/ Hecke action.

Thm (critical FLE)

$$(a) \quad \mathrm{KL}(G)_{\mathrm{crit}} \xrightarrow{\sim} \mathrm{QSh}(\mathcal{O}_{\mathbb{P}^1}^{\mathrm{non-free}})$$

$$DS \xrightarrow{\quad} \downarrow \Gamma \\ \text{Vect}$$

$$\mathcal{O}_{P\check{\sigma}}(D) \subseteq \mathcal{O}_{P\check{\sigma}}^{\text{non-free}} \subseteq \mathcal{O}_{P\check{\sigma}}(D^*)$$

$$\quad \quad \quad \parallel$$

$$\quad \quad \quad \bigsqcup_{\lambda \in \Lambda^+} \mathcal{O}_{P\check{\sigma}}^{\text{non-free}, \lambda}$$

$$\mathcal{O}_{P\check{\sigma}}^{\text{non-free}} = \mathcal{O}_{P\check{\sigma}}(D^*) \times \begin{matrix} LS_{\check{\sigma}}(D) \\ LS_{\check{\sigma}}(D^*) \end{matrix}$$

(b) Compatible w/ Hecke action

$$\begin{matrix} \mathbb{V}^\lambda \\ \text{ind}''(V^\lambda) \end{matrix} \longrightarrow \mathcal{O}_{P\check{\sigma}}^\lambda$$

Cor $\kappa L(\Gamma)_{\text{an}}$ lives over $\mathcal{O}_{P\check{\sigma}}^{\text{non-free}}$

Thm

$$\begin{array}{ccc}
 (a) \text{ } \text{KL}(G)_{\text{cont}, x_1, \dots, x_n} & \xrightarrow{\text{Loc}} & \mathbb{D}(\text{Bur}_G) \\
 \downarrow & \nearrow \widetilde{\text{Loc}} & \\
 \text{KL}(G)_{\text{cont}, x_1, \dots, x_n} \oplus \mathbb{Q}Gh(\mathcal{O}_{\check{\sigma}}^{\text{non-fx}}(X-x_1 \dots x_n)) & & \\
 & \mathbb{Q}Gh(\mathcal{O}_{\check{\sigma}, x_1, \dots, x_n}^{\text{non-fx}}) &
 \end{array}$$

(b) $\widetilde{\text{Loc}}$ is compatible with the action of $\text{Rep}(\check{G})_{\text{Ran}}$.

$$\begin{aligned}
 \text{Rep}(\check{G})_{\text{Ran}} &\xrightarrow{\text{Loc}_{\check{\sigma}, \infty}} \mathbb{Q}Gh(LS_{\check{\sigma}}) \rightarrow \\
 &\rightarrow \mathbb{Q}Gh(\mathcal{O}_{\check{\sigma}}^{\text{non-fx}}(X-x_1 \dots x_n))
 \end{aligned}$$

$$\begin{array}{ccc}
 \text{Whit}(G_G)_{\text{Ran}} & \xrightarrow{\widetilde{\text{FLE}}_{\infty}} & \text{Rep}(\check{G})_{\text{Ran}} \\
 \omega_{\check{\sigma}}^{\text{eff}} \uparrow & & \uparrow \mathbb{P}_{\check{\sigma}, \infty} \\
 \mathbb{D}(\text{Ran}) & \xrightarrow{\text{LL}_G} & \text{THH}(G)
 \end{array}$$

1) $L(G)_{crit}$

$Loc_{G,crit} \uparrow$

$KL(G)_{crit}$

\mathbb{Q}

FLE_{crit}

2) $L(G)_{crit}$

\uparrow $Poinc^{spc}$
Nilp

$QGh(O_{P_i}^{moch})$ Rch

Thm

$O_{P_i}^{moch}(X-x_1 \dots x_n)$

\swarrow
 $O_{P_i}^{moch, x_1 \dots x_n}$

\searrow
 $LS_{P_i}^v$