$N \geq 1$

q is generic (q transsedental)

$$\mathrm{SPerv}_q\left({}_{GL(N-1,\mathcal{O})}\backslash \mathrm{Gr}_{GL(N)}\right) \simeq \mathrm{Rep}_q\left(GL(N-1|N)\right)$$

$K = \mathbb{C}((t)) \supset \mathcal{O} = \mathbb{C}[[t]]$

tensor categories

Also $D_q\left(GL(N-1,\mathcal{O})\backslash \mathrm{Gr}_{GL(N)}\right) \simeq D(\mathrm{Rep}_q \ldots)$

Comments

1. Similar statement exists for any G^s $\quad N > M$

$$\mathrm{Perv}_q\left((H,\chi)\backslash Gr_{GL(N)}\right) \xrightarrow{\sim} \mathrm{Rep}_q(GL(N|M))$$

2. $q = 1$.

$$\mathrm{Perv}\left(GL(N-1,0)\backslash Gr_{GL(N)}\right) \simeq \mathrm{Rep}\left(\underline{GL}(N|N-1)\right)$$

$$\left(\Lambda\left(\mathrm{Hom}(\mathbb{C}^N,\mathbb{C}^{N-1}) \oplus \mathrm{Hom}(\mathbb{C}^{N-1},\mathbb{C}^N)\right)\text{-mod}\right)$$

$GL(N)\times GL(N-1)$ - equiv.

1s

G $\quad$ W - rep. of G

$(\Lambda(W)\text{-mod})^G$

W - symplectic rep.

$W = V \oplus V^* = T^*V$

$\Lambda(V[a] \oplus V^*[b])$

$\circlearrowleft$

$\mathbb{G}_m$ -mod

depends only on $a+b$.

$\{\mathcal{E}$ - G local system on K, s - V-valued flat section$\}$

over $\mathcal{E} = \mathcal{E}_{triv}$ $\quad$ $(V \times V[-1])/G$.

$\mathrm{Sym}(V^*) \otimes \Lambda(V^*[1])$-mod

$\Lambda(V[-1]) \otimes \Lambda(V^*[1])$ -mod

Question How (and when?) can you define a cat. over $LS_{\check{G}}(\operatorname{Spec} K)$ s.t. its fiber over $\mathcal{E}_{\text{triv}}$ is

$(\Lambda^{\cdot}(W)\text{-mod})^{\check{G}}$

$$\operatorname{Perv}_q(GL(N-1,\mathcal{O}) \backslash \operatorname{Gr}_{GL(N)}) \simeq \operatorname{Rep}_q(GL(N|N-1))$$

$GL(N-1,\mathcal{O}) \backslash Gr_{GL(N)}$

Orbits $\longleftrightarrow$ (θ, λ)

dom wt of $GL(N) \times GL(N-1)$

$q=1$ $\quad \mathrm{Rep}(\underline{GL}(N|N-1))$

$(N(K), \chi) \backslash Gr_G$

orbits $\longleftrightarrow$ λ - coweight of G.

Relevant orbits $\longleftrightarrow$ λ dominant

for $q=1$ and the <u>same</u> $\forall q$.

Generic q

Is every orbit relevant?

<u>NO</u> $\qquad \mathrm{Rep}_q(GL(N|N-1))$

Relevant orbits $=$ h. wts

for mixed Borel.

$GL(1) \hookrightarrow GL(2)$

$N=2$

$\mathcal{O}^\times$-orbits in $Gr_{GL(2)}$

$$K/t^i\mathcal{O}$$

θ-dom wt of $GL(2)$

$$\begin{pmatrix} t^\lambda & 0 \\ 0 & 1 \end{pmatrix} \overline{Gr}^\theta \in Gr_{GL(2)}$$

$\lambda \in \mathbb{Z}$

How do you parametrize Irr reps of $GL(N|N-1)$?

Borel $\Leftrightarrow$ flag in $\mathbb{C}^{N|N-1}$

$0 < V_1 < V_2 < V_3 < \dots$

Standard h. wts $\Longleftrightarrow$ dominant wts

Mixed only some dom wts appear

Borel

simple roots are odd

Plan of the proof

1. Interpret $\mathrm{Rep}_q(\dots)$ in terms of some category of factorizable BFS in the usual case

Lurie – very general formalism

2. Try to construct a functor from your geometric category to FS-category — given by an explicit correspondence.

$$\begin{array}{ccc} & Z & \\ \swarrow & & \searrow \\ \mathrm{Gr} & & C^{\cdots} \end{array}$$

$Z \longrightarrow$ Zastava space

C – curve

Usual Zastava and Zastava with poles.

C - curve

$$Z^\theta = \left(\mathrm{Bun}_B^\theta \times \mathrm{Bun}_{N^-}\right)^\circ \longleftarrow \text{transversal at the generic pt of } C.$$

$$\begin{array}{c} \mathrm{Bun}_G \\ \downarrow \pi^\theta \\ C^\theta \end{array}$$

$D \in C^\theta$

$D = D_1 + D_2$ $\quad \deg D_1 = \theta_1$, $\deg D_2 = \theta_2$ disjoint

$$(\pi^\theta)^{-1}(D) = (\pi^{\theta_1})^{-1}(D_1) \times (\pi^{\theta_2})^{-1}(D_2).$$

Zastava with poles

B-bundle $\mathcal{F}_B$

N^--bundle $\mathcal{F}_{N^-}$

$\mathcal{F}_B \sim \mathcal{F}_{N^-}$ on $C \setminus \{c\}$

Gr_G

coloured configurations

$$? \longrightarrow GL(N-1,\mathcal{O})\backslash Gr_{GL(N)}$$

$$GL(N-1) \hookrightarrow GL(N)\times GL(N-1)$$

$$X = GL(N)\times GL(N-1)/GL(N-1) = GL(N)$$

spherical: $B_{GL(N)\times GL(N-1)}$ has an open orbit

Zastava without poles

$\mathcal{E}_N, \mathcal{E}_{N-1}$ - vector bundles on C

$\mathcal{E}_N = \mathcal{E}_{N-1}\oplus\mathcal{O}$

B-structures on $\mathcal{E}_N, \mathcal{E}_{N-1}$ in generic pos.

$$\mathrm{Maps}(C, X/B \supset pt)$$

Fix $c \in C$

$C^{(\mu,\nu)}$

Zastava with poles $\mathcal{E}_N$ is Hecke mod. of $\mathcal{E}_{N-1} \oplus \mathcal{O}$.

$$\mathcal{E}_N \Big|_{C \setminus \{c\}} \simeq \mathcal{E}_{N-1} \oplus \mathcal{O}.$$

(μ,ν)-degree of B-bundles

$GL(N-1,\theta) \backslash Gr_{GL(N)}$

Ind-scheme of divisors

$$D = \sum_{i=1}^{N-1} \delta_i \Delta_i + \sum_{i=1}^{N} \varepsilon_i E_i$$

$\mu \quad \nu$

$$D_{2i} = \Delta_1 + \ldots + \Delta_i - E_1 - \ldots - E_i$$

$$D_{2i-1} = \Delta_1 + \ldots \Delta_{i-1} - E_1 - \ldots - E_i - B$$

effective away from c $B = \sum \Delta_i - \sum E_j$

$$C^{(\mu,\nu)} = \bigcup_{(\lambda,\theta)} C^{(\mu,\nu)}_{\leq(\lambda,\theta)}$$

$D - (\lambda,\theta)c$ is effective everywhere.

$$
\begin{tikzcd}
\mathrm{Gr}_{GL(N)} \arrow[d, dash] & Z^{(\mu,\nu)}_{\le(\lambda,\theta)} \arrow[l, "p"'] \arrow[r, hook, "j"] \arrow[d, "\pi"] & \overline{Z}^{(\mu,\nu)}_{\le(\lambda,\theta)} \arrow[dl, "\overline{\pi}"] \\
GL(N-1,\mathcal{O}) & C^{(\mu,\nu)}_{\le(\lambda,\theta)} &
\end{tikzcd}
$$

Claim 1 (Sakellaridis-Wang)

$\overline{\pi}$ is semi-small.

Claim 2 q-generic

$p^*(?)$ is clean w.r.t j

Claim 3 $p^*(\mathrm{Det}) = \pi^*(\text{explicit})$