Hello

X - curve
Z - target

1) $\mathrm{Maps}^{gen}(X, Z)$

$$\mathrm{Hom}(S, \mathrm{Maps}^{gen}(X, Z))$$
$$\shortparallel$$
$$\left\{ \varphi : \begin{array}{c} U \longrightarrow Z \\ \cap \\ S \times X \end{array} \right.$$

$\forall s \in S \qquad U \cap (s \times X) \underset{\text{dense}}{\subseteq} X$

2) $\mathrm{Maps}^{Ran}(X, Z)$
$$\downarrow$$
$$\mathrm{Ran}$$

$$\mathrm{Hom}(S, \mathrm{Ran}) = \left\{ \begin{array}{l} \text{finite non-empty collections} \\ \text{of elements } \mathrm{Hom}(S, X) \end{array} \right\}$$

$$\mathrm{Hom}(S, \mathrm{Maps}^{\mathrm{Ran}}(X, Z))$$

$$\shortparallel$$

$$\Big\{ \underset{\mathrm{Hom}(S, \mathrm{Ran})}{\underline{x}} \, ; \, \phi : S \times X - \Gamma_{\underline{x}} \longrightarrow Z \Big\}.$$

$$\begin{array}{c} \mathrm{Maps}^{\mathrm{Ran}}(X, Z) \\ \big\downarrow \\ \mathrm{Maps}^{\mathrm{ser}}(X, Z) \end{array}$$

<u>Thm</u> If $Z \subseteq \mathbb{A}^n$ then

$$C_\bullet(\mathrm{Maps}^{\mathrm{ser}}(X, Z)) \longrightarrow \underset{\substack{\shortparallel \\ e}}{C_\bullet(pt)}$$

Drinfeld's proof for the Ran version.

We'll show that the fibers of the projection

$$\begin{array}{c} \mathrm{Maps}^{\mathrm{Ran}}(X, Z) \\ \downarrow \\ \mathrm{Ran} \end{array}$$

are unions (filtered colimits) of prestacks that are $\mathbb{A}^1$-contractible.

Model: $V = \mathrm{Span}\{e_1, e_2, \dots e_n\}$.

Want to show that $V - \{0\}$ is contractible.

$V' = \mathrm{Span}\{e_2, e_3 \dots \}$.

$$(a_1, a_2, a_3, \dots)$$

$$\downarrow$$

$$(t a_1, (1-t)a_1 + t a_2, (1-t)a_2 + t a_3)$$

$$V' - 0 \xrightarrow{id} V' \longrightarrow V$$

$$\Downarrow$$

$$V' - 0 \longrightarrow (1, 0, \dots, 0) \in V$$

$$(0, a_2, a_3, \dots)$$

$$\downarrow$$

$$(t, (1-t)a_2, (1-t)a_3, \dots)$$

$\underline{x} \in \mathrm{Ran}_n \qquad \underline{x} = (x_1, \dots, x_n)$

choose $x = x_1$

$$\mathrm{Maps}(X - \underline{x}, \mathbb{A}^n)_Z \subseteq \mathrm{Maps}(X - \underline{x}, \mathbb{A}^n)$$

maps that generically land in Z.

We'll show that this is contractible.

$$Z = \mathbb{A}^n - V(f)$$

$$f = z_1^d + \sum_{d'<d} z_1^{d'} \tilde{f}_{d'}(z_2, \ldots z_d)$$

N

$$\mathrm{Maps}(X-\underline{x}, \mathbb{A}^n)_{\leq N} \subseteq \mathrm{Maps}(X-\underline{x}, \mathbb{A}^n)$$

$$\mathrm{Maps}(X-\underline{x}, \mathbb{A}^n) = \mathrm{Tot}(\Gamma(X-\underline{x}, \mathcal{O})^{\otimes n})$$

$$\begin{array}{c} \mathrm{Maps}(X-\underline{x}, \mathbb{A}^n)_{\leq N} \cap \mathrm{Maps}(X-\underline{x}, \mathbb{A}^n)_Z \\ \big\downarrow \\ \mathrm{Maps}(X-\underline{x}, \mathbb{A}^n)_Z \end{array}$$

We'll show that $\forall N$ this embedding is homotopic to a constant map.

$N' \geq N \cdot \max_{d'<d} \deg \tilde{f}_{d'}$

$(u_1, \dots u_n) \in \mathrm{Maps}(X - \underline{x}, \mathbb{A}^n)$

If

- $\deg u_i, i \geq 2 \quad \leq N.$
- $f(u_1, \dots u_n) = 0 \Longrightarrow \deg u_1 \leq N'$

deg = order of pole at $x \in \underline{x}$.

$(u_1 \dots \quad u_n) \longrightarrow$

$(1-t)(u_1 \quad \dots u_n) + t(w_t \dots, 0)$

w - some rational function w/ order of pole $> N'$.

$(\tilde{u}_1 \quad \tilde{u}_n)$.

Claim $f(\tilde{u}_1, \dots, \tilde{u}_n) \neq 0$

$$\begin{tikzcd}
\mathrm{Map}^{\mathrm{Ran}}(X, \mathbb{A}^n)_Z \arrow[rr, hook, "j"] \arrow[dr] & & \mathrm{Map}^{\mathrm{Ran}}(X, \mathbb{A}^n) \arrow[dl, "\pi"] \\
& \mathrm{Ran} &
\end{tikzcd}$$

$$C_\cdot(Y) := C^\cdot_c(Y, \omega_Y)$$

$$j_!\left(\omega_{\mathrm{Maps}^{\mathrm{Ran}}(X, \mathbb{A}^n)_Z}\right) \longrightarrow \omega_{\mathrm{Maps}^{\mathrm{Ran}}(X, \mathbb{A}^n)}$$

Thm This map induces an isomorphism after applying $\pi_!$.

I - finite set.

$$\begin{tikzcd}
\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)_Z \arrow[r, "j"] \arrow[dr, "\pi"'] & \mathrm{Maps}^{\times I}(X, \mathbb{A}^n) \\
& \mathrm{Maps}^{\times I}(X, \mathbb{A}^n - Z) \arrow[u, "i"'] \arrow[d, "\pi"] \\
& X^I
\end{tikzcd}$$

$$\begin{tikzcd}
j_!\left(\omega_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)_Z}\right) \arrow[r] & \omega_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)} \arrow[d] \\
& i_! i^* \omega_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)}
\end{tikzcd}$$

Will prove:

$$\pi_! \, i^* \left(\omega_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)}\right) = 0$$

N:

$$\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)_{\le d} \subseteq \mathrm{Maps}^{\times I}(X, \mathbb{A}^n)$$

sum of orders of poles $\le d$ ..

We'll show that

$\pi_! \circ i^*_{\le d}(\omega_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)})$ lives

in cohomological degrees $\le -2d + C$

<u>Proof</u> $\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)_{\le d}$ is

an affine space of dim. $n \cdot d + C$.

Therefore $\omega_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)_{\le d}}$

$$\shortparallel$$

$$e_{\mathrm{Maps}^{\times I}(X, \mathbb{A}^n)}[2nd + C]$$

Claim The fibers of the map

$$\mathrm{Maps}^{X^I}(X, \mathbb{A}^n)_{\leq d} \cap \left(\mathrm{Maps}^{X^I}_s(X, \mathbb{A}^n - Z)\right)$$
$$\downarrow$$
$$X^I$$

are of dimension $\leq (n-1)\cdot d$.

$$-2nd + C + 2(n-1)d = -2d + C.$$

$$\begin{array}{ccc} \mathbb{A}^n & \longrightarrow & \mathbb{A}^{n-1} \\ & & \nearrow \text{finite.} \\ \mathbb{A}^n - Z & & \end{array}$$

Lemma

$$\mathrm{Maps}^{X^I}(X, \mathbb{A}^n - Z) \xrightarrow[\text{finite}]{} \mathrm{Maps}^{X^I}(X, \mathbb{A}^{n-1})$$

grows as $d\cdot(n-1) + \text{const}$.

$$Z \overset{\text{open}}{\subseteq} \underset{\substack{\shortparallel \\ \mathrm{Maps}(X - \underline{x},\, \mathbb{A}^n)}}{\mathbb{A}^\infty} \supseteq V$$

$$\mathbb{A}^\infty = \bigcup_d \mathbb{A}^d$$

$$Z \cap \mathbb{A}^d \hookrightarrow \mathbb{A}^d \hookleftarrow V \cap \mathbb{A}^d$$

Finite-dimensional fact

$$\begin{array}{l} Y \text{ - smooth} \\ \cup\!\shortmid \\ V \text{ - codimension} \geq n. \end{array}$$

$C_\bullet(Y - V) \longrightarrow C_\bullet(Y)$ is

an isomorphism in degrees $\geq -2n$.

$C^\bullet_c(V, \omega_Y|_V)$ live in degree $\leq -2n$.

$$\|$$

$$\omega_Y = \underline{\mathbb{Q}}_\ell[2\dim Y].$$

$$C_c^\bullet(V, \underline{e})\ [2\dim Y].$$

$\mathrm{Bun}_G^{B\text{-gen}}$ – prestack that classifies G-bundles, equipped w/ generic B-reduction

$$\mathrm{Bun}_G^{B\text{-gen}} \to \mathrm{Bun}_G$$

(Jonathan Bar-Lev).

$\mathrm{Hom}(S, \mathrm{Bun}_G^{B\text{-gen}}) =$

$\{\mathcal{P}_G$ on $S \times X$, a reduction of $\mathcal{P}_G$ to B over $U \subseteq S \times X\}$

$\overline{\mathrm{Bun}_B} = \{ \mathcal{M}$ - rank 2 vector bundle on $S \times X$

$\mathcal{L}$ - line bundle on $S \times X$

$\mathcal{L} \xrightarrow{\alpha} \mathcal{M}$ s.t

$\forall s \in S$ $\mathcal{L}_s \longrightarrow \mathcal{M}_s$ is injective as a map of coherent sheaves. $\}$

$$\Updownarrow$$

$\{ \alpha$ is injective and $\mathcal{M}/\mathcal{L}$ is S-flat $\}$

$\mathrm{Bun}_B = \{ \mathcal{M}/\mathcal{L}$ is flat over $S \times X \}$

$\mathrm{Bun}_G^{B\text{-gen}} = (\mathcal{L}, \alpha)$ defined over some U.

i stratification.

$$
\begin{tikzcd}
\mathrm{Bun}_B \arrow[r] \arrow[dr, "p"'] & \overline{\mathrm{Bun}}_B \arrow[r] \arrow[d, "\bar{p}"] & \mathrm{Bun}_G^{B\text{-gen.}} \arrow[dl, "p^{\mathrm{gen}}"] \\
& \mathrm{Bun}_G &
\end{tikzcd}
$$

<u>Lemma</u> $\overline{\mathrm{Bun}}_B$ is proper over Bun_G

$$
\begin{tikzcd}
& \overline{\mathrm{Bun}}_B \underset{\mathrm{Bun}_G^{B\text{-gen.}}}{\times} \overline{\mathrm{Bun}}_B \arrow[dl] \arrow[dr] & \\
\overline{\mathrm{Bun}}_B & & \overline{\mathrm{Bun}}_B
\end{tikzcd}
$$

$$\begin{tikzcd} & \mathrm{Bun}_B \arrow[dl, "p"'] \arrow[dr, "q"] & \\ \mathrm{Bun}_G & & \mathrm{Bun}_T \end{tikzcd}$$

$$\mathrm{Eis}_! = p_! \circ q^*$$

$$D(\mathrm{Bun}_T) \longrightarrow D(\mathrm{Bun}_G)$$

$$\begin{tikzcd} D(\mathrm{Bun}_B) & D(\mathrm{Bun}_G^{B\text{-gen}}) \arrow[l, "i^!"'] \\ D(\mathrm{Bun}_T) \arrow[u, "q^*"] & I(G,B) \arrow[l, "\Gamma"'] \arrow[u, hook, "j_."'] \end{tikzcd}$$

$$\mathrm{Eis}^{enh} : I(G,B) \xrightarrow{p_!^{gen}} D(\mathrm{Bun}_G)$$

$$\mathrm{Eis}_! : D(\mathrm{Bun}_T) \xrightarrow{p_! \circ q^*} D(\mathrm{Bun}_G)$$

$$\mathrm{Bun}_B \xrightarrow{i} \mathrm{Bun}_G^B$$

$$i^! : \mathcal{I}(G,B) \longrightarrow D(\mathrm{Bun}_T)$$

$$i_! \circ q^* : D(\mathrm{Bun}_T) \longrightarrow \mathcal{I}(G,B)$$

$M = i^! \circ i_! \circ q^*$ is a monad on $D(\mathrm{Bun}_T)$

$$M\text{-mod}\,(D(\mathrm{Bun}_T)) = \mathcal{I}(G,B)$$

M acts on $\mathrm{Eis}_!$

$\Downarrow$

$\mathrm{Eis}_!$ can be extended to a functor

$$\mathcal{I}(G,B) \xrightarrow{\mathrm{Eis}^{\mathrm{enh}}} D(\mathrm{Bun}_G)$$