

Hello

X - curve

Z - target

1) $\text{Maps}^{\text{cur}}(X, Z)$

$\text{Hom}(S, \text{Maps}^{\text{cur}}(X, Z))$
"

$\exists \phi : \bigcup_{n_1} \longrightarrow Z$
 $S \times X$

$\forall s \in S \quad \bigcup_{n_1} (s \times X) \subseteq X$
dense

2) $\text{Maps}^{\text{Ran}}(X, Z)$

↓
Ran

$\text{Hoh}(S, \text{Ran}) = \left\{ \begin{array}{l} \text{finite non-empty collections} \\ \text{of elements from } S \times X \end{array} \right\}$

$\text{Hom}(S, \text{Maps}^{\text{Ran}}(X, Z))$
 " "

$\left\{ \frac{x}{n}; \phi: S \times X - \Gamma_x \longrightarrow Z \right\}.$
 $\text{Hoh}(S, \text{Ran})$

$\text{Maps}^{\text{Ran}}(X, Z)$

\downarrow
 $\text{Maps}^{\text{ser}}(X, Z)$

Thm If $Z \subseteq A^n$ then

$C_*(\text{Maps}^{\text{ser}}(X, Z)) \longrightarrow C_*(pt)$
 " "
 e

Drieholz's proof for the Ran version.

We'll show that the fibers of the projection

$$\text{Maps}^{\text{Ran}}(X, Z)$$



Ran

are unions (filtered colimits) of prestacks that are $/A^t$ -contractible.

Model : $V = \text{Span} \{e_1, -e_1, \dots, e_n\}$.

Want to show that $V - \{0\}$ is contractible.

$$V' = \text{Span} \{e_2, e_3, \dots\}.$$

$$(a_1, a_2, a_3, \dots)$$

$$(ta_1, (1-t)a_1 + ta_2, (1-t)a_2 + ta_3)$$

$$\begin{array}{ccc} V'_0 & \xleftarrow{\text{id}} & V' \xrightarrow{\quad} V \\ & \searrow & \downarrow \\ & (1, 0, \dots, 0) \in V & \end{array}$$

$$(0, a_2, a_3, \dots)$$

$$(t, (1-t)a_2, (1-t)a_3, \dots)$$

$$\underline{x} \in \text{Ran } \underline{x} = (x_1, \dots, x_n)$$

choose $x = x_1$

$$\text{Maps}(X - \underline{x}, /A^n) \subseteq \text{Maps}(X - \underline{x}, /A^n)$$

maps that send $n \in Z$.

We'll show that this is contractible.

$$Z = /A^n - V(f)$$

$$f = z_1^1 + \sum_{d' < d} z_1^{d'} \quad \boxed{\tilde{f}_{d'}(z_2, \dots, z_d)}$$

N

$$\text{Maps}(X - \underline{x}, /A^n)_{\leq N} \subseteq \text{Maps}(X - \underline{x}, /A^n)$$

"

$$\text{Tot}(\Gamma(X - \underline{x}, \mathcal{O})^{\otimes n})$$

$$\text{Maps}(X - \underline{x}, /A^n)_{\leq N} \cong \text{Maps}(X - \underline{x}, /A^n)_Z$$



$$\text{Maps}(X - \underline{x}, /A^n)_Z$$

We'll show that $\forall N$ this embedding
is homotopic to a constant map.

$$N' \geq N \cdot \max_{d' < d} \deg \tilde{f}_{d'}$$

$(u_1, \dots, u_n) \in \text{Maps}(X - \underline{x}, \mathbb{A}^n)$

If

- $\deg u_i, i \geq 2 \leq N$.
- $f(u_1, \dots, u_n) = 0 \Rightarrow \deg u_i \leq N'$

$\deg = \text{order of pole at } x \in \underline{x}$.

$(u_1, \dots, u_n) \rightarrow$

$$(1-t)(u_1 \dots u_n) + t(w_t, \dots, 0)$$

w - some rational function of order
of pole $> N'$

$(\tilde{u}_1, \dots, \tilde{u}_n)$.

Claim $f(\tilde{u}_1, \dots, \tilde{u}_n) \neq 0$

$$\text{Map}^{\text{Ran}}(X, A^n)_Z \xrightarrow{j} \text{Map}^{\text{Ran}}(X, A^n)$$

$$\begin{array}{ccc} & \searrow & \downarrow \pi \\ & & R_{\text{can.}} \end{array}$$

$$C_c(Y) := C_c(Y, \omega_Y)$$

$$j_! (\omega_{\text{Map}^{\text{Ran}}(X, A^n)_Z}) \rightarrow \omega_{\text{Map}^{\text{Ran}}(X, A^n)}$$

Thm This map induces an isomorphism
after applying $\pi_!$.

\underline{T} - finite set.

$$\begin{array}{ccc} \text{Maps}^{X^I}(X, A^n)_Z & \xleftarrow{j} & \text{Maps}^{X^I}(X, A^n) \\ & & \xleftarrow{i} \\ & \pi \searrow & \swarrow \pi \\ & X^I & \end{array}$$

$\text{Maps}^{X^I}(X, A^n - Z)$

$$\begin{array}{ccc} j_! (\omega_{\text{Maps}^{X^I}(X, A^n)_Z}) & \rightarrow & \omega_{\text{Maps}^{X^I}(X, A^n)} \\ & & \downarrow \\ & i_* i^* \omega_{\text{Maps}^{X^I}(X, A^n)} & \end{array}$$

Will prove:

$$\pi_! i^* (\omega_{\text{Maps}^{X^I}(X, A^n)}) = 0$$

$N;$

$$\text{Maps}^{X^{\mathbb{P}}}(X, \mathcal{A}^n)_{\leq d} \subseteq \text{Maps}^{X^{\mathbb{P}}}(X, \mathcal{A}^n)$$

\uparrow
sum of orders of poles $\leq d \dots$

We'll show that

$\pi_! \circ i^*_{\leq d}(\omega_{\text{Maps}^{X^{\mathbb{P}}}(X, \mathcal{A}^n)})$ lives
in cohomological degrees $\leq -2d + C$.

Proof $\text{Maps}^{X^{\mathbb{P}}}(X, \mathcal{A}^n)_{\leq d}$ is
an affine space of dim. $n \cdot d + C$.

Therefore $\omega_{\text{Maps}^{X^{\mathbb{P}}}(X, \mathcal{A}^n)_{\leq d}}$
 \cong

$c_{\text{Maps}^{X^{\mathbb{P}}}(X, \mathcal{A}^n)} [2nd + C]$

Claim The fibers of the map

$$\text{Maps}^{X^I}(X, /A^n) \hookrightarrow \cap (\text{Maps}^{X^I}(X, /A^n - z))$$

\downarrow
 X^I

are of dimension $\leq (n-1) \cdot d$.

$$\underline{-2nd + C + 2(n-1)d = -2d + C.}$$

$$\begin{array}{ccc} /A^n & \longrightarrow & /A^{n-1} \\ & \nearrow & \uparrow \text{finite} \\ /A^n - z & & \end{array}$$

Lemmas

$$\text{Maps}^{X^I}(X, /A^n - z) \xrightarrow{\quad} \text{Maps}^{X^I}(X, /A^{n-1})$$

\uparrow
finite

groups \Leftrightarrow
 $d \cdot (n-1) + \text{const.}$

$$Z \subseteq \overset{\text{open}}{\mathbb{A}^\infty} \supseteq V$$

" "

Mors. \$(X - x, \mathbb{A}^n)\$

$$\mathbb{A}^\infty = \bigcup_d \mathbb{A}^d$$

$$Z \cap \mathbb{A}^d \hookrightarrow \mathbb{A}^d \hookleftarrow V \cap \mathbb{A}^d$$

Finite-dimensional fact

\mathcal{J} -smooth

$$\bigvee^{01} V \text{-codimension} \geq n.$$

$C_*(\mathcal{Y} - V) \rightarrow C_*(Y)$ is
an isomorphism in degrees $\geq -2n$.

$C_c(V, \omega_Y|_V)$ live in degrees $\leq -2n$.

||

$$w_Y = \left| \not{d} [2\dim Y] \right|.$$

$$C_c(V, e) [2\dim Y].$$

$Bun_G^{B\text{-gen}}$ - prestack that classifies
 \downarrow G -bundles, equipped w/
generic B -reduction.

Bun_G (Jonathan Bar-Lev).

$$h(S, B_{\mathrm{un}_G^{B\text{-gen}}}) =$$

$\{ \mathcal{F}_U \text{ on } S \times X, \text{ a reduction of } \mathcal{F}_U \}$
 $\text{ s.t. } B \text{ was } U \subseteq S \times X$

Bun_B = { M - rank 2 vector bundle
on $S \times X$

L - like bundle on $S \times X$

$L \xrightarrow{\alpha} M$ s.t

$\forall s \in S \quad L_s \rightarrow M_s$

is injective as a rep
of coherent sheaves.

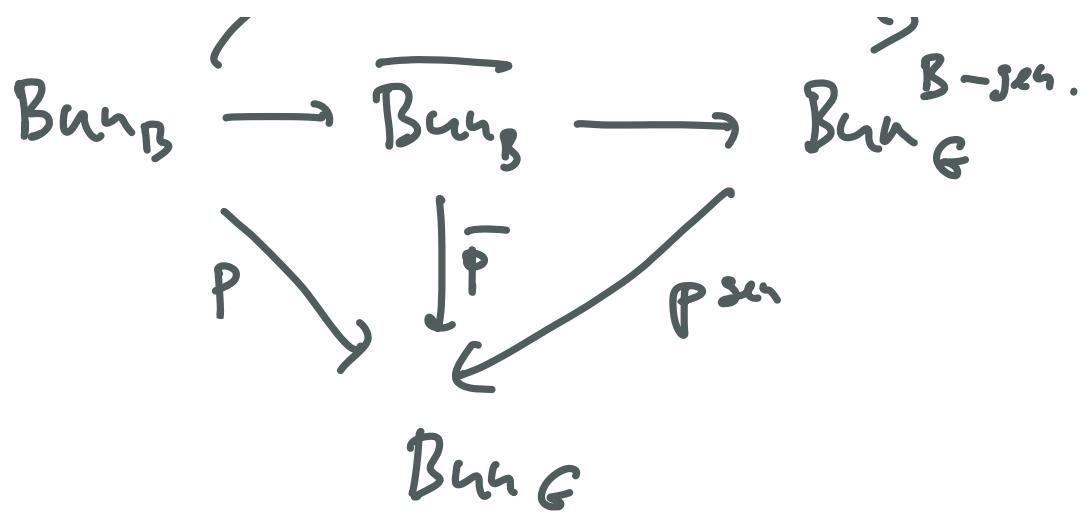


$\{L$ is injective and M/L
is S -flat }

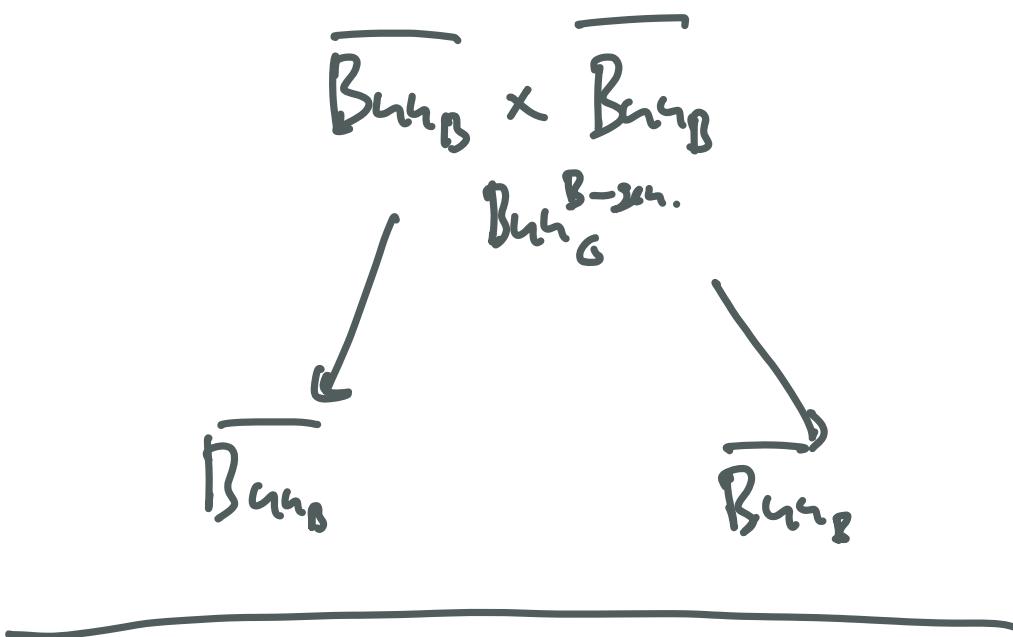
Bun_B = { M/L is flat over $S \times X$ }

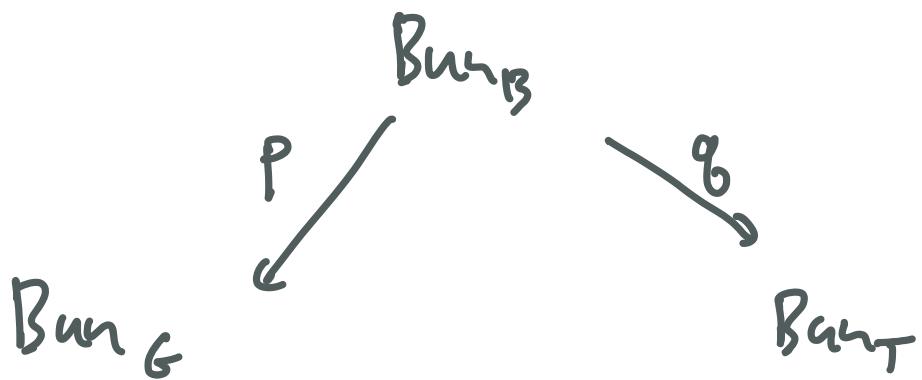
Bun_G^{B-sht} = (L, α) defined over some
 U .

i stratification.



Lem $\overline{\text{Bun}}_B$ is proper over Bun_G





$$\text{Eis}_! = P_! \circ f^*$$

$$D(\text{Bun}_T) \longrightarrow D(\text{Bun}_G)$$

$$D(\text{Bun}_B) \xleftarrow{i^!} D(\text{Bun}_G^{B \rightarrow \infty})$$

$$\begin{array}{ccc}
 & i^! & \\
 \text{Bun}_B & \xleftarrow{\quad g^* \quad} & D(\text{Bun}_G^{B \rightarrow \infty}) \\
 & \uparrow r & \downarrow j \\
 D(\text{Bun}_T) & \xleftarrow{\quad} & I(G, B)
 \end{array}$$

$$\text{Eis}^{\text{enh}}: I(G, B) \xrightarrow{P_!^{\text{enh}}} D(\text{Bun}_G)$$

$$\text{Eis}_!: D(\text{Bun}_T) \xrightarrow{P_! \circ f^*} D(\text{Bun}_G)$$

$$\text{Bun}_B \xrightarrow{i} \text{Bun}_G^B$$

$$i^! : \mathcal{D}(G, B) \longrightarrow \mathcal{D}(\text{Bun}_\tau)$$

$$i_! \cdot q^* : \mathcal{D}(\text{Bun}_\tau) \longrightarrow \mathcal{I}(G, B)$$

$M = i^! \cdot i_! \cdot q^*$ is a monad on
 $\mathcal{D}(\text{Bun}_\tau)$

$$M\text{-nat}(\mathcal{D}(\text{Bun}_\tau)) = \mathcal{P}(G, B)$$

M acts on $Eis!$ ↓

$Eis!$ can be extended to a functor

$$\mathcal{P}(G, B) \xrightarrow{Eis!^\text{ext}} \mathcal{D}(\text{Bun}_G)$$