D. Kaledin & Dario Beraldo

G reductive ~~group~~

$$\begin{array}{c} E \\ \downarrow \\ X \end{array} \qquad B \subseteq G$$

$$\begin{array}{c} E_B \\ \downarrow \\ X \end{array} \qquad E_B \overset{B}{\otimes} G = E$$

$$(E, \nabla_G)$$

$$\begin{array}{c} E_B \\ \downarrow \\ X \end{array} \qquad (E_B)$$

$$B \subseteq G = GL_n$$

$$\begin{array}{c} E \\ \downarrow \\ X \end{array}$$

$$n \subseteq b \subseteq g$$

$$g^{(1)} = [x \in g \mid [x, y] \in b]$$

$$n \subseteq b \subseteq g^{(1)} \subseteq g$$

S

$$
\begin{tikzcd}
0 \arrow[r] & D_{E_B} \arrow[r] \arrow[d] & A_{E_B} \arrow[r] \arrow[d] & T_{X/S} \arrow[r] \arrow[d, equal, "\wr"] & 0 \\
0 \arrow[r] & \mathcal{Y}_E \arrow[r] & A_E \arrow[r] & T_{Y/S} \arrow[r] & 0
\end{tikzcd}
$$

$$
\begin{tikzcd}
T_X \arrow[r, "\nabla"] \arrow[rrr, bend left] \arrow[rrrd, bend right] \arrow[rrrdd, bend right] & A_E \arrow[r] & A_E/A_{E_B} \arrow[r, phantom, "\cong"] & \mathcal{Y}_E/D_{E_B} \\
 & & & \left(\mathcal{Y}^{(1)}/D\right)_E \arrow[u, phantom, "\cup"] \\
 & & & \bigoplus \mathcal{Y}^{(-\alpha)}_E \arrow[u, phantom, "="] \arrow[d] \\
 & & & \left(\mathcal{Y}^{-\alpha}_E\right)
\end{tikzcd}
$$

$F_n \subseteq E$

$\nabla : F_n \to F_{n+1}$

$\nabla : F_n / F_{n-1} \xrightarrow{\theta_{\infty}} F_{n+1} / F_n$

$\rho = (E, \nabla)$:

$\mathcal{O}_{\rho} \subseteq \overline{\mathcal{O}_{\rho}}$

$\mathcal{O}_\rho$, $\overline{\mathcal{O}_\rho}$ into a pre stacks

$\mathcal{O}_\rho(S)$, $\overline{\mathcal{O}_\rho}(S)$ for S an Affine scheme

$\overline{\mathcal{O}_\rho}$ nonempty contains

$U \subset X \times S$ such if the map

$U \to S$ is surjective

$s_0 \in S$

$U_{s_0} \hookrightarrow X$ cofinite image

$$\bar{O}_p(S) = \underline{\operatorname{colim}_{\mathcal{U} \in S \times X} \{ E \times \mathbb{A}^1 \text{ open for } \begin{matrix} \mathcal{U} \\ \downarrow \\ S \end{matrix} \}}$$

$\bar{O}_p$ locally contractible

<u>Thm</u> $\quad X /_k \qquad k(X)$

$$\mathbb{P}^n_{k(X)}$$

$f_1, \dots, f_n \in k(X)[t_0, \dots, t_n]$

d_i deg

$$Z = V(f_1, \dots, f_n) \subseteq \mathbb{P}^n_{k(X)}$$

$$Z(k(X)) \overset{?}{\neq} \emptyset$$

<u>Thm</u> if $\quad \underline{\underline{\sum d_i \leq n \Rightarrow Z(k(X)) \neq \emptyset}}$

$X = \mathbb{P}^1$

$f_1 \dots f_n \in k[X][t_0, \dots, t_n]$

$$\underline{\underline{t_i(X)}} = \sum_{j=0}^{N} t_{ij} X^j$$

$f_i \mid t_0(x)^{g(x)} \ldots t_n(x)^{g(x)} \in \boxed{k[x]} = 0$

$\boxed{d_i N + c_i + 1 = 0}$ $\underline{\underline{g(x)}}$

$f_i = x^2 t_0^2 - t_1^2$

N

$2N + 3$

$(d_i) N + C$

$(N+1) \cdot (n+1)$

$(N+1)(n+1) - 1.$

$\mathbb{P}$

$\underline{\underline{(n+1)N}} + \cancel{n+1}$

$Z(\underline{\underline{k(x)}})$

$Z_{k(x)}$

$$\mathcal{Z} = \operatorname{colim}_N \mathcal{Z}^N_{/k}$$

$$\mathcal{Z}(K(t)) = \mathcal{Z}(K)$$

$$\mathcal{Z}$$

$N-r$

S_r $\mathrm{Sym}^N \mathbb{P}$

$$\mathcal{Z}^N_{/k} \subseteq \mathbb{P}^{N(n+1)+n}_{/k}$$

$$N(\textstyle\sum d_i) + c$$

$$\mathcal{Z}^\infty = \operatorname{colim}_N \mathcal{Z}^N_{/k} \subseteq \operatorname{colim}_N \mathbb{P}^{N(n+1)+n}$$

$$\mathbb{P}^M \times \mathcal{Z}^N \longrightarrow \mathcal{Z}^{N+M}$$

$$\mathbb{P}^\infty \times Z^\infty \longrightarrow Z^\infty$$

$$Z(k(x)) = Z^\infty(k) / \mathbb{P}^\infty(k) \sim \mathbb{P}(k) / \mathbb{P}(k)^{\times}$$

$$Z^\infty \subseteq \mathbb{P}^\infty(k)$$

$$Z^N \longrightarrow \mathbb{P}^{N(l+1)+c}$$

н.у. сущ нрвз

$$Z / k(x) \qquad \bar{Z}$$

Sp_{2n}

$$X \qquad Cl_{2n}$$

$$(E, \nabla)$$

$$\omega : \Lambda^2 E \longrightarrow \mathcal{O}_X$$

$$v \in T_X$$

$$v(\omega(e_1,e_2)) = \omega(\nabla_v(e_1), e_2) + \omega(e_1, \nabla_v(e_2))$$

$$0 = F_0 < F_1 < F_2 < \dots F_n < E$$

$$\mathrm{rank}(F_i) = i$$

$$\omega(F_n, F_n) = 0$$

$$\nabla : F_i \longrightarrow F_{i+1} \otimes \Omega_X$$

Can assume by ~~shrinking~~ X

that $E \simeq \mathcal{O}_X^{\oplus 2n}$ $\quad \Omega_X$ is free

$$\Omega_X \Uparrow T_X = r\mathcal{O}_X$$

$$\nabla_v = v + A : E \longrightarrow E$$

$$A \in \mathfrak{sp}_{2n}(k(X))$$

$$= F_0 < F_2 < \dots < F_n \subseteq \mathcal{O}_X^{\oplus 2n}$$

$$\omega(F_n, F_n) = 0$$

$$\nabla_v(F_i) \subseteq F_{i+1}$$

F_1

$F_2 = \langle F_1, \nabla(F_1) \rangle$

$F_3 = \langle F_1, \nabla(F_1), \nabla^2(F_2) \rangle$

$F_1 \in \mathbb{P}^{2n-1}$

$\omega(F_1, F_1)$

$\omega(\nabla^k(F_1), \nabla^j(F_1)) = 0$

$0 \le (k, j) \le n-1$

$k < j$

$\nabla(\omega(F_1, \nabla(F_1)) = 0)$

$\omega(\nabla F, \cancel{\nabla(F)}) + \boxed{\omega(F, \nabla^2 F) = 0}$

$\omega(\nabla F, \nabla^2 F) + \omega(F, \nabla^3 F) = 0$

$\omega(\nabla^i(F_1), \nabla^{i+1}(F_1)) = 0$

$0 \leq i \leq n-2$

$(n-1)$ quadratic eqn

in $\mathbb{P}^{2n-1}$

$2(n-1) \leq 2n-1$

t_0

$t_0^2 + X t_1' t_2 = 0$

$t_i(x) = \sum t_{ij} x^j$

$t_i' = \sum j t_{ij} x^{j-1}$

$$Op_G^{gen} \to LS_G$$

$$\cup$$

$$\omega_{Op_G^{gen}}$$

$$\Gamma_{X,dR}(\omega_{Op_G}) \in D\text{-}Mod(LS_G)$$

$$St_G$$

$$St_G = \underset{P \in Par}{\mathrm{colim}}\left(\mathrm{colim}(P_!\,\omega_{sp} \to \omega_{LS_G})\right)$$