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A Refinement of a Theorem of J. E. Littlewood

In [**1**], J. E. Littlewood raises the question of how short a doctoral dissertation in mathematics could in principle be,[1] and proposes the precise answer: "Two sentences long." He proves this assertion by pointing out that two sentences is a lower bound, since any thesis in mathematics must contain at least the statement of one theorem and its proof, and then goes on to show that this lower bound could in theory be attained (though it presumably never has been) by exhibiting a one-sentence proof, using only results and notations that were familiar to the mathematicians of the period, of Picard's "small" theorem stating that an entire function avoiding the values 0 and 1 is constant. But Littlewood overlooked the fact that one can shorten the thesis by one whole sentence by placing the statement of the theorem in the title rather than the body of the text (cf. [**2**]). Moreover, his proof of the upper bound is also too complicated, since there is a theorem that is better-known and simpler to prove than Picard's, yet undoubtedly also important enough to justify conferring the title of Doctor of Philosophy on its author. We therefore propose the following corrected and refined version of Littlewood's theorem.

Theorem. *The shortest possible doctoral thesis in mathematics is one sentence long.*

Proof. One sentence is clearly a lower bound, because a thesis in mathematics must contain a proof. We show that this bound can be attained by the following explicit construction of a theoretically possible doctoral thesis.

Bounded entire functions are constant
by Joseph Liouville

submitted to the Department of Mathematics of the University of Paris, 1844,
in partial fulfillment of the requirement of the
Degree of Doctor of Philosophy in Mathematics

If $f(z)$ is bounded and entire, then Mr. Cauchy's theorem implies that

$$f'(a) = \lim_{R\to\infty}\left(\frac{1}{2\pi i}\int_{|z|=R}\frac{f(a+z)}{z^2}\,dz\right) = 0$$

for every a in the complex plane, so $f(z)$ is constant. ■

REFERENCES

1. J. E. Littlewood, *A Mathematician's Miscellany*. Methuen & Co., London, 1953.
2. D. Zagier, A one-sentence proof that every prime $p \equiv 1 \pmod 4$ is a sum of two squares, *Amer. Math. Monthly* **97** (1990) 144.

[1]Actually, Littlewood asks "whether a dissertation of 2 lines could deserve and get a Fellowship," but I have rephrased this in a less Cambridge-oriented way.

—by D. Zagier, Max Planck Institute, Bonn, Germany

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