Proof: By extending the plane to a projective plane and then applying a polarity the problem is reduced to the following:

Suppose we are given a non-concurrent family of lines in the projective plane. The lines are colored red and blue. Prove that there exists a point in the plane which is incident with at least two lines from the family and such that all lines incident with it have the same color.

For sake of brevity, let us call the points incident with at least two lines simply "points", the regions into which our lines divide the projective plane simply "regions", and their corners simply "corner". Suppose indirectly that every point is adjacent to at least one red and at least one blue line. Then at each point there are at least four corners formed by two lines of different colors.

Let r_i denote the number of i-gons among the regions. Since the lines are not all concurrent by assumption, $r_2 = 0$. Let p be the number of points and m the number of line segments. So the number of corners bounded by two differently colored lines is at least 4p. On the other hand, since a triangle has at most two such corners, this number is at most $2r_3 + 4r_4 + 5r_5 + \ldots$.

So

$$4p \leqq 2r_3 + 4r_4 + 5r_5 + \ldots$$

Using Euler's formula for the projective plane

$$m - p + 1 = r_3 + r_4 + r_5 + \ldots$$

and the obvious relation

$$2m = 3r_3 + 4r_4 + 5r_5 + \ldots,$$

we get that for any arrangement of lines,

$$4p = 2r_3 + 4r_4 + 6r_5 + \ldots + 4 > 2r_3 + 4r_4 + 5r_5 + \ldots$$

This contradiction proves the assertion.

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A similar solution to this problem appears in "Sylvester's Problem on Collinear Points and a Relative" by G. D. Chakerian, Amer. Math. Monthly 77, No. 2 (1970). *M. I.*

Problem

A machine emits real numbers at random from the interval (0, 1), one after another.

$f(n)$ is the average least number of numbers emitted whose sum is greater than n.

1. What is $f(1)$? (Hint: e)
2. Show that $f(n) = 2n + \frac{2}{3} + \epsilon_n$ where $\epsilon_n \to 0$ as n increases.
3. Show that the signs of the first sixteen ϵ_n are
   +++−−−++−−−+++−−
   and that this sequence of 16 signs repeats periodically until ϵ_{1328}, which is positive.

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