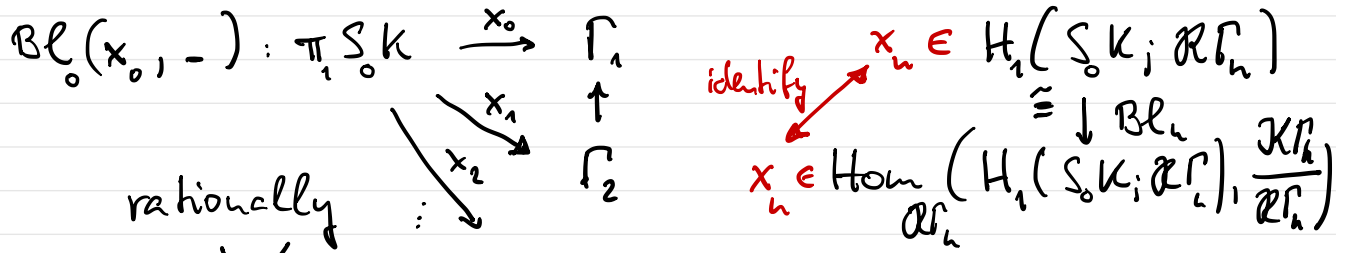


July 17 : Filtrations of the knot concordance group

Recall the def. of $\Gamma_0 = \mathcal{R}$, $\Gamma_{n+1} := \mathcal{R}\Gamma_n / \mathcal{R}\Gamma_n \times \Gamma_n$



Def.: K is \checkmark n -solvable, $n \in \frac{1}{2} \cdot \mathbb{N}_0$, if

- $n = 0$: \exists spin-4-ubd. W_0 s.t. $H_1(W_0)$ freely gen. by m_k : x_{-1} extends
- $n = \frac{1}{2}$: $-\text{"}- S_0 K \xrightarrow{j_{\frac{1}{2}}} W_{\frac{1}{2}}$ $-\text{"}-$ giving Lagrangian $\text{Ker } j_{\frac{1}{2}}$ of $BL_0 \Rightarrow \sigma_{\Gamma_0}^{(n)}(W_{\frac{1}{2}}, \hat{x}_{-1}^0) = 0$
- $n = 1$: $-\text{"}- S_0 K \xrightarrow{j_1} W_1$ $-\text{"}- \forall x_0 \in \text{Ker } j_1 \stackrel{L_1}{=} \text{the map } x_0 \text{ extends to } \pi_1 W_1 \xrightarrow{\hat{x}_0} \Gamma_1$
- $n = 1.5$: $-\text{"}- W_{1.5}$ $-\text{"}-$ giving Lagr. $L_2(x_0)$ for $BL_1(x_0) \forall x_0 \in L_1 \Rightarrow \sigma_{\Gamma_1}^{(n)}(W_{1.5}, \hat{x}_0) = 0$
- etc. see COT 1.

It is clear that K slice $\Rightarrow K$ $\overset{\text{rat.}}{V}$ -solvable
 and that in the genus 1 case $\forall n \in \frac{1}{2} \cdot \mathbb{N}_0$
 we really proved that $\sigma_{\mathbb{Z}}^{(n)}(\mu) \neq 0 \Rightarrow K$ is not
 1.5-solvable.

We actually get a filtration of the knot concordance
 group \mathcal{C} that can be further enhanced in 2 steps.

Def.: K is n -solvable if \exists n -solution W

with $\partial W = S_0 K$, $H_1 W$ freely gen by m_K

$n \in \mathbb{N}_0$: $H_2(W^{(n)}) \ni \langle L_1, \dots, L_2, D_1, \dots, D_n \rangle$ freely gen. $H_2 W$.

n -K derived cover, equiv. intersection form is hyperbolic

$n \in \frac{1}{2} \mathbb{N} - \mathbb{N}$: same, except that L_i are $(n + \frac{1}{2})$ -surfaces
 D_i are $(n - \frac{1}{2})$ -surfaces
 (n) -surfaces, lift by def. to $W^{(n)}$.

Thm: $\forall n \in \frac{1}{2} \cdot \mathbb{N}_0$ and K a knot

(0) K 0-solvable \Leftrightarrow $\text{Arf } K = 0$

($\frac{1}{2}$) K $\frac{1}{2}$ -solvable \Leftrightarrow K alg. slice

(1.5) K 1.5-solvable \Rightarrow all Casson-Gordon invariants of K vanish

(i) K n -solvable \Rightarrow K rat. n -solvable

(ii) K bounds a symmetric grope of height

$(n+2)$ in $\mathbb{D}^4 \Rightarrow K$ is n -solvable

Moreover, $\forall n \in \mathbb{N} \exists$ knots (∞ indep. in \mathcal{C})

that bound $(n+2)$ gropes in \mathbb{D}^4 but are not

(rationally?) n -solvable, see Cochran-T. Finally:

Lemma: \exists Whitney-tower interpretations of (ii)