Smooth points are all alike; every singular point is singular in its own way.

0.1. A smooth point.

\[ x = 0 \in X = \mathbb{C}^n \]

with a torus \( T \) acting linearly with characters \( \chi_1, \ldots, \chi_n \). Then

\[
\dim X = n \quad e_x X = \frac{1}{\chi_1 \chi_2 \cdots \chi_n}.
\]

The cohomology \( H^\bullet(X \setminus \{x\}; \mathbb{Z}) \) is given by:

\[
\begin{array}{cccccc}
H^0 & H^1 & H^2 & \ldots & H^{2n-2} & H^{2n-1} \\
\mathbb{Z} & 0 & 0 & \ldots & 0 & \mathbb{Z}
\end{array}
\]

0.2. A Kleinian singularity.

\[ x = 0 \in X = \text{Kleinian surface singularity of type } A_n \]

\[ = \mathbb{C}^2/\mu_{n+1} = \{(u, v, w) \mid uv = w^{n+1}\} \]

\( T = (\mathbb{C}^*)^2 \) acts on \( X \) via \((\lambda_1, \lambda_2) \cdot (u, v, w) = (\lambda_1 \lambda_2^0 u, \lambda_1^{-1} \lambda_2 v, \lambda_2 w)\).

\[
\dim X = 2 \quad e_x X = \frac{n+1}{(e_1 + ne_2)(e_2 - e_1)}
\]

\((e_1 \text{ and } e_2 \text{ are characters of } T \text{ given by } e_i(\lambda_1, \lambda_2) = \lambda_i.\)

The cohomology \( H^\bullet(X \setminus \{x\}; \mathbb{Z}) \) is given by:

\[
\begin{array}{cccc}
H^0 & H^1 & H^2 & H^3 \\
\mathbb{Z} & 0 & \mathbb{Z}/(n+1) & \mathbb{Z}
\end{array}
\]
0.3. A minimal singularity.

\[ x = 0 \in X = \overline{O}_{\text{min}} \subset \mathfrak{sp}_{2n} \]

If \( T \subset \text{Sp}(2n) \) denotes a maximal torus then \( T \times \mathbb{C}^* \) acts on \( X \) by conjugation and scaling and we can write \( X(T \times \mathbb{C}^*) = X(T) \oplus \mathbb{Z}a_0 \) where \( a_0 \) is the identity character of \( \mathbb{C}^* \). Let \( R_{\text{long}} \subset X(T) \) denote the long roots.

\[
\dim X = 2n
\]

\[
e_x X = \frac{2^{2n-1}}{\prod_{\alpha \in R_{\text{long}}} (\alpha + a_0)}
\]

The cohomology \( H^\bullet(X \setminus \{x\}; \mathbb{Z}) \) is given by:

\[
\begin{array}{cccccccc}
H^0 & H^1 & H^2 & H^3 & H^4 & \ldots & H^{4n-3} & H^{4n-2} & H^{4n-1} \\
\mathbb{Z} & 0 & \mathbb{Z}/\langle 2 \rangle & 0 & \mathbb{Z}/\langle 2 \rangle & \ldots & 0 & \mathbb{Z}/\langle 2 \rangle & \mathbb{Z}
\end{array}
\]

0.4. A minimal degeneration.

\( G \): be a simple algebraic group of type \( G_2 \)

\( G_G \): affine Grassmannian of \( G \)

\( \varpi_1^\vee, \varpi_2^\vee \): fundamental coweights (regarded as points of \( G_G \))

\( \overline{G}_{\varpi_2^\vee} \): Schubert variety indexed by \( \varpi_2^\vee \)

\[
X := (L^{<0}G \cdot \varpi_1^\vee) \cap \overline{G}_{\varpi_2^\vee}
\]

Then \( X \) is a \( \overline{T} \)-variety where \( \overline{T} = T \times \mathbb{C}^* \) denotes the extended torus, and \( \varpi_1^\vee \) is a attractive \( \overline{T} \)-fixed point.

Then \( X \) is of (complex) dimension 4 and we have

\[
e_x X = \frac{27}{(\alpha_0 + \alpha_1)(\alpha_0 + \alpha_1 + 3\alpha_2)(2\alpha_0 + 5\alpha_1 + 6\alpha_2)(2\alpha_0 + 5\alpha_1 + 9\alpha_2)}.
\]

The cohomology \( H^\bullet(X \setminus \{x\}; \mathbb{Z}) \) is given by:

\[
\begin{array}{cccccccc}
H^0 & H^1 & H^2 & H^3 & H^4 & H^5 & H^6 & H^7 \\
\mathbb{Z} & 0 & \mathbb{Z}/\langle 3 \rangle & 0 & \mathbb{Z}/\langle 3 \rangle & 0 & \mathbb{Z}/\langle 3 \rangle & \mathbb{Z}
\end{array}
\]