Before: How to draw/encode morphisms. Now: how to draw/encode elements. Choose RHS, find basis of BS(\(w\)) as right R-mod. Size is \(2^{\ell(x)}\). A description:

**Description 1:** 01-sequences. Consider BS. We know \(101\) are right R-basis.

**Better basis:**
\[ C_4 = 101 \text{ convoled} \]
\[ C_3 = \frac{1}{2} (498 + 1000) \text{ also convoled} \]
\[ f_{C_3} = f (\frac{1}{2} (498 + 1000)) = C_3. \]

**Remember:**
\[ f_1 = f (\frac{1}{2} (498 + 1000)) \]
\[ f_{C_3} = f (\frac{1}{2} (498 + 1000)) \]

Consider BS\(_{\mathbb{R}}\)... BS\(_{\mathbb{R}}\):

\[ \rho \in \mathbb{F}_p^{\ell(\mathbb{F}_p)} \]
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**Exercise:** Write \( FC_{C_3} \) in right R-basis for \( f \) linear.

More properties of BS\(_{\mathbb{R}}\):
- BS\(_{\mathbb{R}}(\mathbb{W})\) is a commutative ring.
- \( \mathbb{R} \otimes_{\mathbb{R}} \mathbb{R} \text{ tensor mlt.} \)
- \( \text{grading is wrong though } b/c \ C_{\mathbb{W}} = \mathbb{I} \text{ dim deg - d.} \)
- \( \text{deg}(a-b) = \text{deg} a + \text{deg} b - \text{deg} c. \)

In 01 sequences, mlt = stacking, i.e.,
\[ \rho \in (C_{\mathbb{W}}) \] \[ \rho \in (C_{\mathbb{W}}) \]
\[ \rho \in (C_{\mathbb{W}}) \]

**Ex:**
\[ \begin{array}{c|c}
1 & 0 \\
0 & 1 \\
\end{array} \]
\[ \begin{array}{c|c}
0 & 1 \\
1 & 0 \\
\end{array} \]
\[ \begin{array}{c|c}
1 & 0 \\
0 & 1 \\
\end{array} \]

BS\(_{\mathbb{R}}\) also has a trace \( \text{Tr} : \text{BS(}\mathbb{W}) \longrightarrow \mathbb{R} \) take coeff of \( C_\rho \) (this is canonical...)

and a pairing \( \text{BS(}\mathbb{W}) \times \text{BS(}\mathbb{W}) \longrightarrow \mathbb{R} \)

\[ \langle a, b \rangle = \text{Tr}(ab) \]

**Ex:**
\[ \langle 1, 0 \rangle = 0 \]
\[ \langle 0, 1 \rangle = 0 \]

**Claim:**
\[ \langle C_{\rho}, C_{\rho} \rangle = 1 \text{ and } \langle C_{\rho}, C_{\rho} \rangle = 0 \text{ for } \rho \neq \rho \text{ in some order.} \]

\[ \Rightarrow \text{non-degenerate to intersect 0.} \]

This pairing is invariant:
\[ \text{deg}(a \circ b) = \text{deg} a + \text{deg} b \]
\[ \langle a \circ b, c \rangle = \langle a, c \rangle \text{ if } (a \circ b) \cup c \text{ (nothing)} \]
Some abstract bullshit: \[ \text{Hom}_{\text{Right } R-\text{mod}}(B, R) = DB \] 

\[ \text{Hom}(B, DB)^0 = \text{space of invariant forms on } B \]

\[ \text{R-Bim} \]

\[ \text{isom} \Leftarrow \text{nondeg to degree 0} \]

This BS(\(\omega\)) has nondeg form \(\Rightarrow\) BS(\(\omega\)) is self-dual. What about other SBim? 

By Exercise: \(R_w\) still bim is self-dual. 

Thus \(SCT \Rightarrow B_w \in \text{BS}(\omega)\) so \(DB_w \otimes \text{BS}(\omega)\), ! property \(\Rightarrow B_w \leq DB_w\) 

\[ \text{All ineq a hom are self-dual} \Rightarrow \exists \text{ nondeg invt form} \]

What is this form? Clearly \(B_w\) inherits a form from BS(\(\omega\)). 

Exercise: \(C_{\text{top}} \in B_w\) (use support filtration). Thus restricted form is non-zero. 

Cor: Suppose S. Conj, so that End(\(B_w\)) = R. Then any non-zero invt form is the unique nondeg invt form (up to \(R^x\)). 

We'll prove this en route to proving S. Conj latter. Example to come.

Second description of elts of BS(\(\omega\)), also parametrized by 2nd term...

Any elt is \(\Psi(C_{\text{bot}})\) for some \(\Psi \in \text{End}(\text{BS}(\omega))\). Well, \(\Psi = \sum_{x} w \wedge f_{w} \cdot \omega \)

Prop: For each \(x\) appearing as a subexp. of \(\omega\), \(\exists! \) subexp \(c_{\text{bot}, x}\) s.t. 

\[ LL_{\text{can}} x: \text{BS}(\omega) \rightarrow \text{BS}(x) \]

\(c_{\text{bot}, x}\) to some \(y\) nonzero!

It has maximal degree, and is minimal in the Bruhat path dominance order.

Moreover, \(LL_{\text{can}} x(C_{\text{bot}, \omega}) = C_{\text{bot}, x}\)

Pf: By map sends \(c_{\text{bot}} \rightarrow C_{\text{bot}}\), so must have only \(y\)s!

Exercise \(\Rightarrow\) the rest of the prop.

\[
\begin{array}{ccc}
\omega_0 & \rightarrow & \omega_2 \\
\omega_1 & \rightarrow & \omega_2 \\
\end{array}
\]
**Remark:** \( L_{can} \) depends on a choice of res move, but two choices differ by a term which kills \( C_{bot} \).

\[
\sum_{can} \frac{LL}{x} \cdot f(C_{bot}) = \sum_{can} \frac{LL}{x} \cdot f(C_{bot}, x)
\]

Upward light lemmas are injective, and finally given by \( Z \).

\[
\begin{align*}
\text{In this description what is multiplication?} & \quad \frac{LL}{x} \cdot \frac{LL}{y} = ? \\
\text{Not obvious!} & \quad \text{CBS to description?}
\end{align*}
\]

**One nice fact:**

**Claim:** If \( \Psi(C_{bot}) = 0 \), then coeff of \( \frac{LL}{x} \) is zero, i.e., it's in lower term now.

\[
\Psi(C_{bot}) = \sum (\text{coeff of } \frac{LL}{x}) \cdot \text{symbol } \frac{LL}{x}
\]

Perhaps, this coeff is zeros.

\[\text{Will use later.}\]

**TIME FOR QUESTIONS**