Soergel bimodules and Kazhdan-Lusztig conjectures

QGM, Aarhus
March 2013

Friday problem sheet

Research level questions:

Please let us know if you begin seriously thinking about or working on these problems. It is best to avoid overlap and competition.

1. Find a geometric interpretation of the inductive argument for the Hodge-Riemann bilinear relations using Rouquier complexes.

2. Apply similar techniques to decompose objects in other algebras related (possibly) to geometry. The prime example is quiver Hecke algebras (i.e. KLR algebras). You will certainly need the **relative** Hodge-Riemann bilinear relations.

3. Find a subset of light leaves (in description 2) for a Bott-Samelson $BS(\underline{w})$ of a reduced expression that projects to a basis of the indecomposable bimodule B_w. (This would be an analog of Littelmann's path model for intersection cohomology.)

4. Find a combinatorial formula for the local intersection form of two light leaves, or perhaps for other morphisms. (See the earlier question we asked: make light leaves canonical.)

5. Calculate the p-canonical basis (or the p-linkage classes) in some interesting new cases.

6. Develop the theory of p-left cells, etc.

7. Develop the computational ability to find the lower terms appearing in the type H_3 Zamolodzhikov relation. Alternatively, develop the theory which explains what lower terms appear when comparing two rex moves in general.

8. For W finite, with longest element having a reduced expression $\underline{w}_0$, what is the idempotent in $BS(\underline{w}_0)$ projecting to B_{w_0}? This is known in type A; similar arguments clearly suffice in type B; type H is impossible without the previous question; but even types D and E would be interesting.

9. Consider the monoidal category generated by $R \otimes_{R^s} R$, where $s \in T$ is a reflection. (Recall that T is the set of all conjugates of S in W.) What is the Grothendieck group of this category? This is unknown even in type A_2. (Speak to Anne-Laure Thiel if you are interested in this problem.)

10. Describe the 2-category $\mathrm{Fund}_{\mathfrak{g}}$ by generators and relations, for an arbitrary complex semisimple lie algebra $\mathfrak{g}$. (Don't think this is easy. But singular Soergel bimodules are a new tool.)

11. Describe quantum algebraic Satake outside of type A. The lack of monodromy in the affine Cartan matrix prevents one from deforming it as in type A, although this might work in type C.

12. Give a geometric explanation for quantum algebraic Satake. Is there any connection to the work of Gaitsgory?

13. Describe the categorification of the J-ring for cells in arbitrary Coxeter groups.

14. What Hodge-theoretic statements can be made about other kinds of Rouquier complexes (both positive and negative, not reduced, etc)?

15. Use Hodge-theoretic properties to aid the computation of triply graded knot homology. (Vivek Shende is surely interested here.)